SCIENTIFIC MEMOIRS

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VIII.

THE EFFECTS OF A MAGNETIC FIELD ON RADIATION

THE EFFECTS

OF

A MAGNETIC FIELD ON RADIATION

MEMOIRS BY FARADAY, KERR
AND ZEEMAN

EDITED BY

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PREFACE

Historical

In the early part of this century possible relationships between the various "forces of nature" began to attract the attention of physicists. In 1800 William Herschel discovered that a "heat spectrum" is superimposed on and extends beyond the visible solar spectrum, indicating some relationship between heat and light. This seems to have suggested to Domenico Morichini, of Rome, the search for a relationship between light and magnetism. In 1812 he claimed that he had been able to magnetize steel needles by exposing them to the violet radiation in the solar spectrum. Others, including Mrs. Somerville, in England, believed that they had verified his results, but many were unable to reproduce them, and it was finally demonstrated that all these effects had been due to other causes. The dispute over this question extended over many years, and is an instructive illustration of the difficulty which even skilled experimenters may have in solving a comparatively simple experimental problem.

About 1825 Sir John Herschel sent a polarized beam of light along the axis of a helix carrying an electric current. Examination with an analyzer showed no effect. He also intended to test the effect of a polarized beam passing tangentially by a conductor carrying a current, but never executed the experiment.

No other attempt to show a relationship between light and magnetism seems to have been made until Faraday undertook the investigation described in the following pages.

Theoretical

In the *Proceedings of the Royal Society* for June, 1856, Sir William Thomson wrote: "The magnetic influence on light

discovered by Faraday depends on the direction of moti
moving particles. For instance, in a medium possessi
particles in a straight line parallel to the lines of mag
force, displaced to a helix round this line as axis, and
projected tangentially with such velocities as to de
circles, will have different velocities according as thei
tions are round in one direction (the same as the no
direction of the galvanic current in the magnetizing co
in the contrary direction. But the elastic reaction o
medium must be the same for the same displacements,
ever be the velocities and directions of the particles; tl
to say, the forces which are balanced by centrifugal fo
the circular motions are equal, while the luminiferous m
are unequal. The absolute circular motions being, ther
either equal, or such as to transmit equal centrifugal for
the particles initially considered, it follows that the lum
rous motions are only components of the whole motion
that a less luminiferous component in one direction,
pounded with a motion existing in the medium when
mitting no light, gives an equal resultant to that of a g
luminiferous motion in the contrary direction, compo
with the same non-luminous motion."

Maxwell, in his *Electricity and Magnetism*, vol. ii., cha
offers the following partial physical explanation as an
sion of the above remarks:* "It is a well-known theor
kinematics that two uniform circular vibrations, of the
amplitude, having the same periodic time, and in the
plane, but revolving in opposite directions, are equiv
when compounded together, to a rectilinear vibration.
periodic time of this vibration is equal to that of the ci
vibrations, its amplitude is double, and its direction is i
line joining the points at which two particles, describin
circular vibrations in opposite directions around the
circle, would meet. . . . We may therefore express the
nomenon of the rotation of the plane of polarization i
following manner:—A plane-polarized ray falls on the me
This is equivalent to two circularly polarized rays, one
handed, the other left-handed (as regards the observer).

* In 1855 Verdet suggested a similar explanation. (*Ann. Chim*
43, p. 37, 1856.)

passing through the medium the ray is still plane-polarized, but the plane of polarization is turned, say, to the right (as regards the observer). Hence, of the two circularly polarized rays, that which is right-handed must have had its phase accelerated with respect to the other during its passage through the medium.

"In other words, the right-handed ray has performed a greater number of vibrations, and therefore has a smaller wave-length, within the medium, than the left-handed ray which has the same periodic time. . . . From this we conclude, from the reasoning of art. 21, that in the medium, when under the action of magnetic force, some rotatory motion is going on, the axis of rotation being in the direction of the magnetic forces; and that the rate of propagation of circularly polarized light, when the direction of its vibratory rotation and the direction of the magnetic rotation of the medium are the same, is different from the rate of propagation when these directions are opposite.

"This angular velocity cannot be that of any portion of the medium of sensible dimensions rotating as a whole. We must, therefore, conceive the rotation to be that of very small portions of the medium, each rotating on its own axis. This is the hypothesis of molecular vortices.

"The motion of these vortices, though, as we have shown, it does not sensibly affect the vibratory motions of large bodies, may be such as to affect that vibratory motion on which the propagation of light, according to the undulatory theory, depends. The displacements of the medium during the propagation of light will produce a disturbance of the vortices, and the vortices, when so disturbed, may react on the medium so as to affect the mode of propagation of the ray.

"It is impossible, in our present state of ignorance as to the nature of the vortices, to assign the form of the law which connects the displacement of the medium with the variation of the vortices."

Righi proved experimentally that a right-handed circularly polarized beam travels more rapidly than a left-handed one in substances which cause a right-handed rotation in a magnetic field.

The physical explanation of the problem is complicated by the fact that the magnetic force does not affect the ethereal

vibrations directly, but only through the intervention of matter.

Reference to some of the theories will be made in discussing the Kerr effect. Several of the theories lead to an expression for the rotation of the form

$$\theta = \mathrm{MH}l\frac{\mu^2}{\lambda^2}\left(\mu - \lambda\frac{d\mu}{d\lambda}\right),$$

where M is a constant depending on the medium, H the intensity of the field, l the thickness of the medium, μ its index of refraction, and λ the wave-length of the light. This expression is in fair accord with the results of experiments.

The decomposition of a linear vibration into two circular components travelling with different velocities in a magnetized medium will account for the Faraday effect, but the Kerr effect is much too complicated to be explained by such a simple lation.

E. H. Hall * in 1880 discovered that the stream lines of an electric current flowing through a thin conducting sheet transverse to a magnetic field are deflected, indicating the existence of a small "magnetic component" at right angles to the original current and the field. Rowland, assuming that a similar effect exists in a dielectric medium, showed that such an effect would account for rotation. Bassett, H. A. Lorentz, and others † have likewise explained the Kerr effect in an analogous manner, but in each case the explanation was incomplete in some point; moreover, the Hall effect itself was left unexplained.

Lorentz assumed that all electrical disturbances in dielectrics are due to the motions of charged "dielectric ions" (entirely different from electrolytic ions), which are subject to ponderomotive forces when moving in a magnetic field. If the anions and cations, the motion of which in opposite directions constitutes an electric current, move with equal velocities, they will be equally displaced by the field, and there can be no electrical separation.

Wind assumed that there are "conductive" as well as "dielectric" ions, and that the oppositely charged ions move

* [*Phil. Mag.* (5), 10, 136, 1880; *Am. Jl. Science* (3), 20, p. 52, 1880.]

† [*See Bibliography.*]

with unequal velocities. There is, therefore, a resultant separation at right angles to the current and to the field, which explains the Hall effect.

The various theories based on the Hall effect give an expression for the electromotive intensity in a magnetic field of the form

$$E=\frac{i}{p}-qHi,$$

i being the current strength, H the intensity of the field, and p and q constants depending on the medium and the frequency of vibration. The last term represents the Hall effect.

By assuming p and q to be complex instead of real quantities, Wind has deduced from the above equation a perfect explanation of the Kerr effect.

If this physical explanation be correct, the coefficient of the Hall effect cannot be the same for rapidly oscillating as for steady currents. In iron and cobalt the rotation of transmitted light is in the same direction as in nickel, but the coefficients of the Hall effect are of opposite sign.

Several other more or less satisfactory theories have been advanced, references to some of which are given in the Bibliography.

Later Investigations of the Faraday Effect

All of the many substances tested by Faraday and other early investigators, except doubly refracting crystals and gases, were found to possess the power of rotating the plane of polarization in a magnetic field, and in every case the rotation was in the nominal direction of the magnetizing current. Later, it was found that all gases, and doubly refracting crystals under certain conditions, also possess this property; so likewise do iron, nickel, and cobalt in the form of thin transparent films.

In 1846 E. Becquerel discovered that the rotation in a given substance varies, very nearly, inversely as the square of the wave-length of the light employed. One of the effects of this magnetic rotatory dispersion had been noted by Faraday.* In some substances, such as tartaric acid, the rotation is anomalous.

* [*See p.* 6.]

In 1852 Verdet began a long series of observations of magnetic rotation, in which he verified the following laws, which had been more or less explicitly stated by Faraday and others.

1. The rotation θ for a given wave-length is directly proportional to the length of path in the medium.

2. It is proportional to the resolved part of the intensity of the field in the direction of the beam, or

$$\theta = \omega H l.$$

More generally, if V_1 and V_2 be the magnetic potentials at the opposite boundaries of the medium

$$\theta = \omega (V_1 - V_2),$$

Verdet's constant ω depends on the nature of the medium.

3. The rotation for different colors is nearly inversely as the square of the wave-length.

4. In solutions the rotations of the components are algebraically additive.

In examining solutions of perchloride of iron Verdet found that it produced a negative rotation, which in strong solutions was sufficient to overcome the positive rotation of the water used as a solvent. As this salt is paramagnetic, it was for some time assumed that the rotation in diamagnetic substances is always positive, that in paramagnetic substances negative. Subsequently it was found that no such simple relations exist—that, for example, paramagnetic nickel and cobalt salts in solution, and iron, nickel, and cobalt in thin films, rotate positively, while diamagnetic titanium chloride rotates negatively. The only general rule that seems to hold is that given by Kundt—that all elementary substances produce a positive rotation.

In 1879 Kundt and Röntgen and H. Becquerel succeeded independently in detecting and measuring the rotation produced by gases. Kundt and Röntgen studied most of the ordinary gases under pressures as high as 250 atmospheres, and found that in every case the rotation was positive and directly proportional to the density. Becquerel used gases at ordinary pressure, and employed Faraday's device of multiple reflection to increase the effect.

L. H. Siertsema has recently made more exact measurements

of the magnetic rotation and dispersion in air, oxygen, nitrogen, hydrogen, and other gases. In every case he found positive rotation and normal dispersion. The rotation can be represented by the formula

$$\theta = \frac{A}{\lambda}\left(1 + \frac{B}{\lambda^2}\right).$$

Kerr's discovery in 1877 of the rotation of the plane of polarization of light reflected from a magnet suggested that thin films of the magnetic metals would probably rotate the plane of polarization of transmitted light. In 1884 Kundt succeeded in depositing iron, nickel, and cobalt on glass in such thin films as to transmit light. In each case a large positive rotation was produced in a magnetic field. In the case of iron, the rotation was some 30,000 times that produced by an equal thickness of glass, and nearly 1500 times greater than the natural rotation of quartz. The rotation was apparently proportional to the thickness of the film, and for small-field intensities proportional to the magnetic force. For stronger fields the rotation increased less rapidly than the field, and finally reached a maximum of about 200,000° per centimeter thickness, in the case of iron. Kundt inferred that the rotation is proportional to the intensity of magnetization, not to the magnetizing force. Du Bois repeated Kundt's experiments, and verified this assumption. If κ be the susceptibility of the substance, V the magnetic potential, and ϕ the potential of magnetization,

$$\phi = \kappa V,$$

$$\theta = \omega(V_1 - V_2) = \frac{\omega}{\kappa}(\phi_1 - \phi_2) = \psi(\phi_1 - \phi_2).$$

ψ is called Kundt's constant.

The rotation in different media bears no simple relation to their relative susceptibilities. Cobalt has almost as strong rotative power as iron, but that of nickel is less than half as great.

Kundt observed that the rotation of iron is anomalous, red light being rotated more than violet. Lobach found that the dispersion in iron, nickel, and cobalt is anomalous throughout the visible spectrum, the rotation always increasing with the wave-length.

Perkin, Schönrock, Jahn, Humburg, and others have studied the rotation produced by organic liquids and by solutions of

acids and salts. The rotation produced by isomeric substances is not the same. In homologous series, the increased rotation produced by each additional radical is approximately the same. In inorganic solutions the molecular rotation is in many cases practically constant for all degrees of concentration, which indicates that dissociation does not influence rotation to any appreciable extent.

Efforts have been made to find some relation between the magnetic rotation and the index of refraction of a substance. H. Becquerel found that in many cases the ratio $\frac{\theta}{\mu^2(\mu^2-1)}$ is constant; but in general this does not hold. There is likewise no apparent relation between natural dispersion and magnetic rotatory dispersion.

In 1846 E. Becquerel found a slight magnetic rotation of a beam of polarized light passing through certain double-refracting crystals parallel to their principal axes. Wertheim (1852), Lüdtge (1869), Chauvin (1886), and Wedding (1888) investigated the phenomena in natural crystals and glass under stress. Gouy (1888) developed a theory, based on the simple superposition of displacements due to magnetic rotation and to the two linear vibrations of the polarized components, which indicated that the rotation in a double-refracting medium should have alternate maxima and minima as the length of path in the medium increases, passing through zero for a difference of path of the two components of $\frac{1}{2}\lambda$. This was verified by Wedding, who found no rotation for a difference of path of $\frac{1}{2}\lambda$ in glass under stress. For greater differences the rotation became negative, reaching a maximum negative value for a difference of $\frac{3}{4}\lambda$. The rotation rapidly falls off as the polarized beam becomes inclined to the axis, passes through a series of diminishing maxima and minima, and vanishes when the inclination reaches a few degrees.

Faraday assumed that unpolarized light is likewise subject to rotation, and this was demonstrated by Sohncke in 1886. Homogeneons unpolarized light fell on two parallel slits close to each other, each pencil passing through a bar of glass surrounded by a coil of wire. The interference bands formed on a screen beyond showed a distinct loss of visibility when a current traversed the coils, indicating that the simultaneous directions of corresponding vibrations in the two wave-fronts had

become inclined to each other, thus weakening the parallel components, which alone are capable of interference.

The effect of changes in temperature on magnetic rotation is small. In general there is a slight diminution as the temperature rises, and this diminution does not in general vary directly as the change in density.

Later Investigations of the Kerr Effect

The Kerr effect consists, in general terms, of (1) a rotation of the plane of polarization of the reflected light, and (2) the change of plane into elliptically polarized light, this being due to a new "magnetic component" at right angles to the incident vibration and having a different phase. The first effect may be attributed to the rotatory power of a thin surface layer of the metal through which the light penetrates before reflection. Poincaré and others believed this effect due to the rotating power of the adjacent air, but Kerr's conclusion that the rotation was produced neither before nor after reflection, but during the process, was verified by Fitzgerald and by Kundt. The former covered the magnetic mirror with gold-leaf and the latter deposited on it electrolytically a very thin non-magnetic film. In these cases no rotation was produced.

In 1881 Hall found that the plane of polarization of light reflected from magnetized nickel and cobalt is rotated in the same direction as by iron.

Important investigations have been made by Righi, Kundt, Du Bois, Sissingh, Zeeman, and others. Their results may be summarized as follows:

Polar reflection with normal incidence.—The predominant effect is one of rotation. The reflected light is elliptically polarized, but the effect is very small.

Polar reflection with oblique incidence.—When the plane of polarization is perpendicular to the plane of incidence (or the vibrations in that plane), the reflected light is rotated, and very slightly elliptically polarized, the rotation reaching a maximum for an angle of incidence between 44° and 68°. When the plane of polarization coincides with the plane of incidence, the rotation is less (steadily diminishing as the angle of incidence increases), and the ellipticity greater. In all cases of polar reflection the rotation seems to be negative, or in the opposite

direction to that of the magnetizing current, although some of Kundt's results for iron seem to show a slight positive rotation for angles of incidence greater than 80°.

Equatorial reflection with normal incidence.—In this case there is no rotation.

Equatorial reflection with oblique incidence, and the plane of incidence parallel to the field.—The rotation of light polarized parallel to the plane of incidence is always negative, reaching a negative maximum at an angle of incidence of about 65°, for iron. When the plane of polarization is perpendicular to the plane of incidence the rotation is at first positive, reaches a maximum at an angle of incidence of about 65°, for iron, then diminishes and changes sign, at about 80° for iron, 50° to 60° for nickel, and 78° for cobalt. The reflected light is elliptically polarized in all cases except where the incident light is polarized parallel to the plane of incidence and is incident at the angle giving maximum rotation.

Equatorial reflection, with plane of incidence perpendicular to the field.—It was believed that in this case no rotation was produced. Wind's theory led, however, to the conclusion that light polarized perpendicularly to the plane of incidence must be slightly rotated and elliptically polarized. This conclusion was experimentally verified by Zeeman.

Du Bois found that magnetite gives a positive rotation, which is apparently the same for all faces of the crystal.

The rotation approaches a maximum with increasing field strength, and Du Bois has shown that, as in the case of the Faraday effect, the rotation is proportional to the intensity of magnetization. In general, for normal incidence, $\theta = K\kappa H_n = KI_n$, where H_n and I_n are the normal components of the intensity of field and the intensity of magnetization respectively, κ the coefficient of susceptibility, and K a constant. This is additional proof that the rotation takes place within the metal.

The rotatory dispersion of iron, nickel, cobalt, and magnetite is anomalous.

Later Investigations of the Zeeman Effect

Michelson has analyzed the spectral lines of various metals in a magnetic field by his interferential method. He found usually a more complicated structure of the lines than can be shown by the diffraction grating.

In general, the separation of the components varies directly as the intensity of the field, and (for homologous lines) inversely as the square of their wave-length. Lines belonging to different homologous series are affected very differently.

In the case of some quadruplets observed at right angles to the field, Cornu found that the central pair of components were polarized perpendicularly to the force-lines, the outer pair parallel to them.

Ames, Earhart, Reese, and Becquerel found that in the case of some iron lines the polarization of the middle and outer components was reversed in direction. Preston considers that this may be due to the greater separation of the components of the middle line, causing them to overlap the outer components. The doubling of the central components he explains as being possibly due to a periodic variation of the amplitude of vibration. The tripling of lines may in general be due to a precessional motion of the elliptic orbits of the ions, and doubling of each component of the triplet (in the case of sextets) by an apsidal motion of the orbit.

Becquerel and Deslandres confirmed Zeeman's observation that no effect is produced by the magnetic field on band spectra. This is in accordance with the view that such spectra belong to a complicated molecular structure, not to independent ions.

The Editor takes this opportunity to thank Prof. Zeeman for his kindness in furnishing material and revising the proofs of his memoirs.

GENERAL CONTENTS

ON THE MAGNETIZATION OF LIGHT AND THE ILLUMINATION OF MAGNETIC LINES OF FORCE

BY

MICHAEL FARADAY, F. R. S.

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A

CONTENTS

ON THE MAGNETIZATION OF LIGHT AND THE ILLUMINATION OF MAGNETIC LINES OF FORCE *

BY

MICHAEL FARADAY, F. R. S.

I. ACTION OF MAGNETS ON LIGHT

2146.† I HAVE long held an opinion, almost amounting to con-

* The title of this paper has, I understand, led many to a misapprehension of its contents, and I therefore take the liberty of appending this explanatory note. Neither accepting nor rejecting the hypothesis of an ether, or the corpuscular or any other view that may be entertained of the nature of light; and, as far as I can see, nothing being really known of a ray of light more than of a line of magnetic or electric force, or even of a line of gravitating force, except as it and they are manifest in and by substances; I believe that, in the experiments I describe in the paper, light has been magnetically affected, *i. e.*, that that which is magnetic in the forces of matter has been affected, and in turn has affected that which is truly magnetic in the force of light. By the term magnetic I include here either of the peculiar exertions of the power of a magnet, whether it be that which is manifest in the magnetic or the diamagnetic class of bodies. The phrase "illumination of the lines of magnetic force" has been understood to imply that I had rendered them luminous. This was not within my thought. I intended to express that the line of magnetic force was illuminated as the earth is illuminated by the sun, or the spider's web illuminated by the astronomer's lamp. Employing a ray of light, we can tell, *by the eye*, the direction of the magnetic lines through a body; and by the alteration of the ray and its optical effect on the eye can see the course of the lines just as we can see the course of a thread of glass, or any other transparent substance, rendered visible by the light; and this was what I meant by *illumination*, as the paper fully explains.—December 15, 1845. M. F.

† [*The numbers at the beginning of the paragraphs refer to those in the Experimental Researches.*]

viction, in common, I believe, with many other lovers of natural knowledge, that the various forms under which the forces of matter are made manifest have one common origin; or, in other words, are so directly related and mutually dependent that they are convertible, as it were, one into another, and possess equivalents of power in their action.* In modern times the proofs of their convertibility have been accumulated to a very considerable extent, and a commencement made of the determination of their equivalent forces.

2147. This strong persuasion extended to the powers of light, and led, on a former occasion, to many exertions having for their object the discovery of the direct relation of light and electricity and their mutual action in bodies subject jointly to their power,† but the results were negative, and were afterwards confirmed, in that respect, by Wartmann.‡

2148. These ineffectual exertions, and many others which were never published, could not remove my strong persuasion derived from philosophical considerations; and, therefore, I recently resumed the inquiry by experiment in a most strict and searching manner, and have at last succeeded in *magnetizing and electrifying a ray of light and in illuminating a magnetic line of force.* These results, without entering into the detail of many unproductive experiments, I will describe as briefly and clearly as I can.

2149. But before I proceed to them I will define the meaning I connect with certain terms which I shall have occasion to use:—thus, by *line of magnetic force*, or *magnetic line of force*, or *magnetic curve*, I mean that exercise of magnetic force which is exerted in the lines usually called magnetic curves, and which equally exist as passing from or to magnetic poles, or forming concentric circles round an electric current. By *line of electric force*, I mean the force exerted in the lines joining two bodies acting on each other according to the principles of static electric induction (1161, etc.), which may also be either in curved or straight lines. By a *diamagnetic*, I mean a body through which lines of magnetic force are passing, and which does not by their action assume the usual magnetic state of iron or loadstone.

* *Exp. Res.*, 57, 366, 376, 877, 961, 2071.

† *Phil. Trans.*, 1834. *Exp. Res.*, 951-955. [*Efforts to discover effects of electricity on polarized light.*]

‡ *Archives de l'Électricité*, ii., pp. 596–600.

A MAGNETIC FIELD ON RADIATION

2150. A ray of light issuing from an Argand lamp was polarized in a horizontal plane by reflection from a surface of glass, and the polarized ray passed through a Nicol's eye-piece revolving on a horizontal axis, so as to be easily examined by the latter. Between the polarizing mirror and the eye-piece two powerful electromagnetic poles were arranged, being either the poles of a horse-shoe magnet or the contrary poles of two cylinder magnets; they were separated from each other about two inches in the direction of the line of the ray, and so placed that, if on the same side of the polarized ray, it might pass near them; or, if on contrary sides, it might go between them, its direction being always parallel, or nearly so, to the magnetic lines of force (2149). After that, any transparent substance placed between the two poles would have passing through it both the polarized ray and the magnetic lines of force at the same time and in the same direction.

2151. Sixteen years ago I published certain experiments made upon optical glass * and described the formation and general characters of one variety of heavy glass, which, from its materials, was called silicated borate of lead. It was this glass which first gave me the discovery of the relation between light and magnetism, and it has power to illustrate it in a degree beyond that of any other body. For the sake of perspicuity I will first describe the phenomena as presented by this substance.

2152. A piece of this glass, about two inches square and 0.5 of an inch thick, having flat and polished edges, was placed in a *diamagnetic* (2149) between the poles (not as yet magnetized by the electric current), so that the polarized ray should pass through its length; the glass acted as air, water, or any other indifferent substance would do; and if the eye-piece were previously turned into such a position that the polarized ray was extinguished, or rather the image produced by it rendered invisible, then the introduction of this glass made no alteration in that respect. In this state of circumstances the force of the electromagnet was developed by sending an electric current through its coils, and immediately the image of the lamp flame became visible, and continued so as long as the arrangement continued magnetic. On stopping the electric current, and so causing the magnetic force to cease, the light instantly disap-

* *Phil. Trans.*, 1830, p. 1.

peared. These phenomena could be renewed at pleasure, at any instant of time, and upon any occasion, showing a perfect dependence of cause and effect.

2153. The voltaic current which I used upon this occasion was that of five pair of Grove's construction, and the electro-magnets were of such power that the poles would singly sustain a weight of from twenty-eight to fifty-six, or more, pounds. A person looking for the phenomenon for the first time would not be able to see it with a weak magnet.

2154. The character of the force thus impressed upon the diamagnetic is that of *rotation;* for when the image of the lamp flame has thus been rendered visible, revolution of the eye-piece to the right or left, more or less, will cause its extinction; and the further motion of the eye-piece to the one side or other of this position will produce the reappearance of the light, and that with complementary tints, according as this further motion is to the right or left hand.

2155. When the pole nearest to the observer was a marked pole, *i. e.*, the same as the north end of a magnetic needle, and the further pole was unmarked, the rotation of the ray was right-handed; for the eye-piece had to be turned to the right hand, or clock fashion, to overtake the ray and restore the image to its first condition. When the poles were reversed, which was instantly done by changing the direction of the electric current, the rotation was changed also and became left-handed, the alteration being to an equal degree in extent as before. The direction was always the same for the same *line of magnetic force.*

2156. When the diamagnetic was placed in the numerous other positions which can easily be conceived about the magnetic poles, results were obtained more or less marked in extent and very definite in character, but of which the phenomena just described may be considered as the chief example; they will be referred to, as far as is necessary, hereafter.

2157. The same phenomena were produced in the silicated borate of lead by the action of a good ordinary steel horse-shoe magnet, no electric current being now used. The results were feeble, but still sufficient to show the perfect identity of action between electromagnets and common magnets in this their power over light.

2158. Two magnetic poles were employed end-ways, *i. e.*, the cores of the electromagnets were hollow iron cylinders, and the ray of polarized light passed along their axes and through the diamagnetic placed between them: the effect was the same.

2159. One magnetic pole only was used, that being one end of a powerful cylinder electromagnet. When the heavy glass was beyond the magnet, being close to it, but between the magnet and the polarizing reflector, the rotation was in one direction, dependent on the nature of the pole; when the diamagnetic was on the near side, being close to it but between it and the eye, the rotation for the same pole was in the contrary direction to what it was before; and when the magnetic pole was changed both these directions were changed with it. When the heavy glass was placed in a corresponding position to the pole, but above or below it, so that the *magnetic curves* were no longer passing through the glass parallel to the ray of polarized light, but rather perpendicular to it, then no effect was produced. These particularities may be understood by reference to Fig. 1, where *a* and *b* represent the first positions of the diamagnetic, and *c* and *d* the latter positions, the course of the ray being marked by the dotted line. If, also, the glass were placed directly at the end of the magnet, then no effect was produced on a ray passing in the direction here described, though it is evident, from what has been already said (2155), that a ray passing *parallel* to the magnetic lines through the glass so placed would have been affected by it.

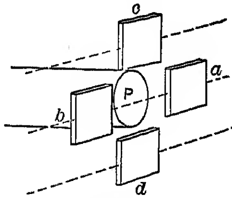

Fig. 1

2160. Magnetic lines, then, in passing through silicated borate of lead, and a great number of other substances (2173), cause these bodies to act upon a polarized ray of light when the lines are parallel to the ray, or in proportion as they are parallel to it. If they are perpendicular to the ray, they have no action upon it. They give the diamagnetic the power of rotating the ray; and the *law* of this action on light is, that if a magnetic line of force be *going from* a north pole or *coming* from a south pole, along the path of a polarized ray coming to the observer, it will rotate that ray to the right hand; or, that if such a line of force be coming from a north pole or

going from a south pole, it will rotate such a ray to the left hand.

2161. If a cork or a cylinder of glass, representing the diamagnetic, be marked at its ends with the letters N and S, to represent the poles of a magnet, the line joining these letters may be considered as a magnetic line of force; and, further, if a line be traced round the cylinder with arrow heads on it to represent direction, as in the figure [2], such a simple model, held up before the eye, will express the whole of the law, and give every position and consequence of direction resulting from it. If a watch be considered as the diamagnetic, the north pole of a magnet being imagined against the face, and a south pole against the back, then the motion of the hands will indicate the direction of rotation which a ray of light undergoes by magnetization.

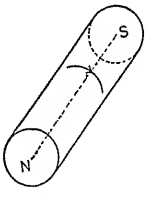

Fig. 2

2162. I will now proceed to the different circumstances which affect, limit, and define the extent and nature of this new power of action on light.

2163. In the first place, the rotation appears to be in proportion to the extent of the diamagnetic through which the ray and the magnetic lines pass. I preserved the strength of the magnet and the interval between its poles constant, and then interposed different pieces of the same heavy glass (2151) between the poles. The greater the extent of the diamagnetic in the line of the ray, whether in one, two, or three pieces, the greater was the rotation of the ray; and, as far as I could judge by these first experiments, the amount of rotation was exactly proportionate to the extent of diamagnetic through which the ray passed. No addition or diminution of the heavy glass on the *side* of the course of the ray made any difference in the effect of that part through which the ray passed.

2164. The power of rotating the ray of light *increased* with the intensity of the magnetic lines of force. This general effect is very easily ascertained by the use of electromagnets; and, within such range of power as I have employed, it appears to be directly proportionate to the intensity of the magnetic force.

2165. Other bodies, besides the heavy glass, possess the same power of becoming, under the influence of magnetic force,

active on light (2173). When these bodies possess a rotative power of their own, as is the case with oil of turpentine, sugar, tartaric acid, tartrates, etc., the effect of the magnetic force is to add to or subtract from their specific force, according as the natural rotation and that induced by the magnetism is right or left handed (2231).

2166. I could not perceive that this power was affected by any degree of motion which I was able to communicate to the diamagnetic whilst jointly subject to the action of the magnetism and the light.

2167. The interposition of copper, lead, tin, silver, and other ordinary non-magnetic bodies in the course of the magnetic curves, either between the pole and the diamagnetic, or in other positions, produced no effect either in kind or degree upon the phenomena.

2168. Iron frequently affected the results in a very considerable degree; but it always appeared to be either by altering the direction of the magnetic lines or disposing within itself of their force. Thus, when the two contrary poles were on one side of the polarized ray (2150), and the heavy glass in its best position between them and in the ray (2152), the bringing of a large piece of iron near to the glass on the other side of the ray caused the power of the diamagnetic to fall. This was because certain lines of magnetic force, which at first passed through the glass parallel to the ray, now crossed the glass and the ray; the iron giving two contrary poles opposite the poles of the magnet, and thus determining a new course for a certain portion of the magnetic power, and that across the polarized ray.

2169. Or, if the iron, instead of being applied on the opposite side of the glass, were applied on the same side with the magnet, either near it or in contact with it, then, again, the power of the diamagnetic fell, simply because the power of the magnet was diverted from it into a new direction. These effects depend much, of course, on the intensity and power of the magnet, and on the size and softness of the iron.

2170. The electro-helices (2190) without the iron cores were very feeble in power, and, indeed, hardly sensible in their effect. With the iron cores they were powerful, though no more electricity was then passing through the coils than before (1071). This shows, in a very simple manner, that the

phenomena exhibited by light under these circumstances is directly connected with the magnetic form of force supplied by the arrangement. Another effect which occurred illustrated the same point. When the contact at the voltaic battery is made, and the current sent round the electromagnet, the image produced by the rotation of the polarized ray does not rise up to its full lustre immediately, but increases for a couple of seconds, gradually acquiring its greatest intensity; on breaking the contact, it sinks instantly and disappears apparently at once. The gradual rise in brightness is due to the *time* which the iron core of the magnet requires to evolve all that magnetic power which the electric current can develop in it; and as the magnetism rises in intensity, so does its effect on the light increase in power; hence the progressive condition of the rotation.

2171. I cannot as yet find that the heavy glass (2151), when in this state, *i. e.*, with magnetic lines of force passing through it, exhibits any increased degree or has any specific magneto-inductive action of the recognized kind. I have placed it in large quantities, and in different positions, between magnets and magnetic needles, having at the time very delicate means of appreciating any difference between it and air, but could find none.*

2172. Using water, alcohol, mercury, and other fluids contained in very large delicate thermometer-shaped vessels, I could not discover that any difference in volume occurred when the magnetic curves passed through them.

2173. It is time that I should pass to a consideration of this power of magnetism over light as exercised, not only in the silicated borate of lead (2151), but in many other substances; and here we perceive, in the first place, that if all transparent bodies possess the power of exhibiting the action, they have it in very different degrees, and that up to this time there are some that have not shown it at all.

2174. Next, we may observe that bodies that are exceedingly different to each other in chemical, physical, and mechanical

* [*In a later series of experiments Faraday discovered that practically all bodies have paramagnetic or diamagnetic properties.*]

properties develop this effect; for solids and liquids, acids, alkalies, oils, water, alcohol, ether, all possess the power.

2175. And, lastly, we may observe that in all of them, though the degree of action may differ, still it is always the same in kind, being a rotative power over the ray of light; and, further, the direction of the rotation is, in every case, independent of the nature or state of the substance, and dependent upon the direction of the magnetic line of force, according to the law before laid down (2160).

[2176 *omitted.*]

2177. Whilst employing crystalline bodies as diamagnetics, I generally gave them that position in which they did not affect the polarized ray, and then induced the magnetic curves through them. As a class, they seemed to resist the assumption of the rotating state. *Rock-salt* and *fluorspar* gave evidence of the power in a slight degree; and I think that a crystal of alum did the same, but its ray length in the transparent part was so small that I could not ascertain the fact decisively. Two specimens of transparent fluor, lent me by Mr. Tennant, gave the effect.

2178. Rock crystal, four inches across, gave no indications of action on the ray, neither did smaller crystals, nor cubes about three-fourths of an inch in the side, which were so cut as to have two of their faces perpendicular to the axis of the crystal, though they were examined in every direction.

2179. *Iceland spar* exhibited no signs of effect, either in the form of rhomboids or of cubes like those just described.

2180. *Sulphate of baryta, sulphate of lime,* and *carbonate of soda* were also without action on the light.

2181. A piece of fine clear *ice* gave me no effect. I cannot, however, say there is none, for the effect of water in the same mass would be very small, and the irregularity of the flattened surface from the fusion of the ice and flow of water made the observation very difficult.

2182. With some degree of curiosity and hope, I put gold-leaf into the magnetic lines, but could perceive no effect. Considering the extremely small dimensions of the length of the path of the polarized ray in it, any positive result was hardly to be expected.

[*Par.* 2183–2185, *describing experiments on many fixed and essential oils and aqueous solutions, are omitted.*]

2186. Proceeding from liquids to air and gaseous bodies, I have here to state that, as yet, I have not been able to detect the exercise of this power in any one of the substances in this class. I have tried the experiment with bottles four inches in diameter and the following gases: oxygen, nitrogen, hydrogen, nitrous oxide, olefiant gas, sulphurous acid, muriatic acid, carbonic acid, carbonic oxide, ammonia, sulphuretted hydrogen, and bromine vapor, at ordinary temperatures; but they all gave negative results. With air the trial has been carried, by another form of apparatus,* to a much higher degree, but still ineffectually.

[2187, 2188 *omitted.*]

II. ACTION OF ELECTRIC CURRENTS ON LIGHT.

2189. From a consideration of the nature and position of the lines of magnetic and electric force, and the relation of a magnet to a current of electricity, it appeared almost certain that an electric current would give the same result of action on light as a magnet; and, in the helix, would supply a form of apparatus in which great lengths of diamagnetics, and especially of such bodies as appeared to be but little affected between the poles of the magnet, might be submitted to examination and their effect exalted. This expectation was, by experiment, realized.

[2190–2220 *omitted.*]

III. GENERAL CONSIDERATIONS.

2221. Thus is established, I think for the first time,† a true,

* [*See pp.* 24, 25.]

† I say for the first time, because I do not think that the experiments of Morichini on the production of magnetism by the rays at the violet end of the spectrum prove any such relation. When in Rome with Sir H. Davy in the month of May, 1814, I spent several hours at the house of Morichini, working with his apparatus and under his directions, but could not succeed in magnetizing a needle. I have no confidence in the effect as a *direct* result of the action of the sun's rays; but think that when it has occurred it has been secondary, incidental, and perhaps even accidental; a result that might well happen with a needle that was preserved during the whole experiment in a north and south position.

direct relation and dependence between light and the magnetic and electric forces; and thus a great addition made to the facts and considerations which tend to prove that all natural forces are tied together, and have one common origin (2146). It is, no doubt, difficult in the present state of our knowledge to express our expectation in exact terms; and, though I have said that another of the powers of nature is, in these experiments, directly related to the rest, I ought, perhaps, rather to say that another form of the great power is distinctly and directly related to the other forms, or that the great power manifested by particular phenomena in particular forms is here further identified and recognized, by the direct relation of its form of light to its forms of electricity and magnetism.

2222. The relation existing between *polarized* light and magnetism and electricity is even more interesting than if it had been shown to exist with common light only. It cannot but extend to common light, and as it belongs to light made, in a certain respect, more precise in its character and properties by polarization, it collates and connects it with these powers in that duality of character which they possess, and yields an opening, which before was wanting to us, for the appliance of these powers to the investigation of the nature of this and other radiant agencies.

2223. Referring to the conventional distinction before made (2149), it may be again stated that it is the magnetic lines of force *only* which are effectual on the rays of light, and they *only* (in appearance) when parallel to the ray of light, or as they tend to parallelism with it. As, in reference to matter not magnetic after the manner of iron, the phenomena of electric induction and electrolyzation show a vast superiority in the energy with which electric forces can act as compared with magnetic forces, so here, in another direction and in the peculiar and correspondent effects which belong to magnetic forces, they are shown, in turn, to possess great superiority, and to have their full equivalent of action on the same kind of matter.

2224. The magnetic forces do not act on the ray of light di-

January 2, 1846.—I should not have written "for the first time," as above, if I had remembered Mr. Christie's experiments and papers "On the Influence of the Solar Rays on Magnets," communicated in the *Philosophical Transactions* for 1826, p. 219, and 1828, p. 379.—M. F

rectly and without the intervention of matter, but through the mediation of the substance in which they and the ray have a simultaneous existence, the substances and the forces giving to and receiving from each other the power of acting on the light. This is shown by the non-action of a vacuum, of air or gases; and it is also further shown by the special degree in which different matters possess the property. That magnetic force acts upon the ray of light always with the same character of manner and in the same direction, independent of the different varicties of substance, or their statcs of solid or liquid, or their specific rotative force (2232), shows that the magnetic force and light have a direct relation; but that substances are necessary, and that these act in different degrees, shows that the magnetism and the light act on each other through the intervention of the matter.

2225. Recognizing or perceiving *matter* only by its powers, and knowing nothing of any imaginary nucleus, abstract from the idea of these powers, the phenomena described in this paper must strengthen my inclination to trust in the views I have on a former occasion advanced in reference to its nature.*

2226. It cannot be doubted that the magnetio forces act upon and affect the internal constitution of the diamagnetic just as freely in the dark as when a ray of light is passing through it; though the phenomena produced by light seem, as yet, to present the only means of observing this constitution and the change. Further, any such change as this must belong to opaque bodies, such as wood, stone, and metal; for as diamagnetics there is no distinction between them and those which are transparent. The degree of transparency can at the utmost, in this respect, only make a distinction between the individuals of a class.

2227. If the magnetic forces had made these bodies magnets, we could, by light, have examined a transparent magnet, and that would have been a great help to our investigation of the forces of matter. But it does not make them magnets (2171), and therefore the molecular condition of these bodies, when in the state described, must be specifically distinct from that of magnetized iron or other such matter, and must be *a new mag-*

* "A Speculation," etc., *Phil. Mag.*, xxiv., p. 136, 1844. [*In this paper Faraday advocated Boscovich's hypothesis that atoms are merely centres of force.*]

netic condition; and as the condition is a state of tension (manifested by its instantaneous return to the normal state when the magnetic induction is removed), so the *force* which the matter in this state possesses, and its mode of action, must be to us a new *magnetic force* or *mode of action* of matter.

2228. For it is impossible, I think, to observe and see the action of magnetic forces, rising in intensity, upon a piece of heavy glass or a tube of water, without also perceiving that the latter acquire properties which are not only new to the sub-
stance, but are also in subjection to very definite a
laws (2160, 2199), and are equivalent in proportion t
netic forces producing them

2229. Perhaps this state
to a current; as in magnets, according to Ampère's theory, the state is a state of *current*. When a core of iron is put into a helix, everything leads us to believe that currents of electricity are produced within it, which rotate or move in a plane perpendicular to the axis of the helix. If a diamagnetic be placed in the same position, it acquires power to make light rotate in the same plane. The state it has received is a state of tension, but it has not passed on into currents, though the acting force and every other circumstance and condition are the same as those which do produce currents in iron, nickel, cobalt, and such other matters as are fitted to receive them. Hence the idea that there exists in diamagnetics, under such circumstances, a tendency to currents is consistent with all the phenomena as yet described, and is further strengthened by the fact that, leaving the loadstone or the electric current, which by inductive action is rendering a piece of iron, nickel, or cobalt magnetic, perfectly unchanged, a mere change of temperature will take from these bodies their extra power and make them pass into the common class of diamagnetics.

2230. The present is, I believe, the first time that the molecular condition of a body required to produce the circular polarization of light has been artificially given; and it is therefore very interesting to consider this known state and condition of the body, comparing it with the relatively unknown state of those which possess the power naturally; especially as some of the latter rotate to the right hand and others to the left; and

as, in the cases of quartz and oil of turpentine, the same body, chemically speaking, being in the latter instance a liquid with particles free to move, presents different specimens, some rotating one way and some the other.

2231. At first one would be inclined to conclude that the natural state and the state conferred by magnetic and electric forces must be the same, since the effect is the same; but on further consideration it seems very difficult to come to such a conclusion. Oil of turpentine will rotate a ray of light, the power depending upon its particles and not upon the arrangement of the mass. Whichever way a ray of polarized light passes through this fluid, it is rotated in the same manner; and rays passing in every possible direction through it *simultaneously* are all rotated with equal force and according to one common law of direction; *i. e.*, either all right-handed or else all to the left. Not so with the rotation superinduced on the *same* oil of turpentine by the magnetic or electric forces: it exists only in one direction, *i. e.*, in a plane perpendicular to the magnetic line; and being limited to this plane, it can be changed in direction by a reversal of the direction of the inducing force. The direction of the rotation produced by the natural state is connected invariably with the direction of the ray of light; but the power to produce it appears to be possessed in every direction and at all times by the particles of the fluid. The direction of the rotation produced by the induced condition is connected invariably with the direction of the magnetic line or the electric current, and the condition is possessed by the particles of matter, but strictly limited by the line or the current, changing and disappearing with it.

[2232, 2233 *omitted.*]

2234. All these differences, however, will doubtless disappear or come into harmony as these investigations are extended; and their very existence opens so many paths by which we may pursue our inquiries, more and more deeply, into the powers and constitution of matter.

2235. Bodies having rotating power of themselves do not seem by that to have a greater or a less tendency to assume a further degree of the same force under the influence of magnetic or electric power.

2236. Were it not for these and other differences we might see an analogy between these bodies, which possess at all times

the rotating power, as a specimen of quartz which rotates only in one plane, and those to which the power is given by the induction of other forces, as a prism of heavy glass in a helix on the one hand; and, on the other, a natural magnet and a helix through which the current is passing. The natural condition of the magnet and quartz, and the constrained condition of the helix and heavy glass, form the link of the analogy in one direction; whilst the supposition of currents existing in the magnet and helix, and only a tendency or tension to currents existing in the quartz and heavy glass, supplies the link in the transverse direction.

2237. As to those bodies which seem as yet to give no indication of the power over light, and, therefore, none of the assumption of the new magnetic conditions, these may be divided into two classes, the one including air, gases, and vapors, and the other rock crystal, Iceland spar, and certain other crystalline bodies. As regards the latter class, I shall give, in the next series of these researches, proofs drawn from phenomena of an entirely different kind that they do acquire the new magnetic condition;* and these being so disposed of for the moment, I am inclined to believe that even air and gases have the power to assume the peculiar state, and even to affect light, but in a degree so small that, as yet, it has not been made sensible. Still, the gaseous state is such a remarkable condition of matter that we ought not too hastily to assume that the substances which, in the solid and liquid state, possess properties even general in character, always carry these into their gaseous condition.

2238. Rock-salt, fluor-spar, and, I think, alum, affect the ray of light; the other crystals experimented with did not; these are equiaxed and singly refracting, the others are unequiaxed and doubly refracting. Perhaps these instances, with that of the rotation of quartz, may even now indicate a relation between magnetism, electricity, and the crystallizing forces of matter.

2239. All bodies are affected by helices as by magnets, and according to laws which show that the causes of the action are identical as well as the effects. This result supplies another fine proof in favor of the identity of helices and magnets, according to the views of Ampère.

* [*So far as concerns mechanical forces in a magnetic field.*]

2240. The theory of static induction which I formerly ventured to set forth (1161, etc.), and which depends upon the action of the contiguous particles of the dielectric intervening between the electric and the inducteous bodies, led me to expect that the same kind of dependence upon the intervening particles would be found to exist in magnetic action; and I published certain experiments and considerations on this point seven years ago (1709–1736). I could not then discover any peculiar condition of the intervening substance or diamagnetic; but now that I have been able to make out such a state, which is not only a state of tension (2227), but dependent entirely upon the magnetic lines which pass through the substance, I am more than ever encouraged to believe that the view then advanced is correct.

2241. Although the magnetic and electric forces *appear* to exert no power on the ordinary or on the depolarized ray of light, we can hardly doubt but that they have some special influence, which probably will soon be made apparent by experiment; neither can it be supposed otherwise than that the same kind of action should take place on the other forms of radiant agents as heat and chemical force.

2242. This mode of magnetic and chemical action, and the phenomena presented by it, will, I hope, greatly assist hereafter in the investigation of the nature of transparent bodies, of light, of magnets, and their action one on another, or on magnetic substances. I am at this time engaged in investigating the new magnetic condition, and shall shortly send a further account of it to the Royal Society. What the possible effect of the force may be in the earth as a whole, or in magnets, or in relation to the sun, and what may be the best means of causing light to evolve electricity and magnetism, are thoughts continually pressing upon the mind; but it will be better to occupy both time and thought, aided by experiment, in the investigation and development of real truth than to use them in the invention of suppositions which may or may not be founded on or consistent with fact.

ROYAL INSTITUTION, *Oct.* 29, 1845.

ON THE MAGNETIC AFFECTION OF LIGHT

BY

MICHAEL FARADAY, F. R. S.

(*Philosophical Magazine,* Series 3, xxix., pp. 153, 249, 1846)

CONTENTS

ON THE MAGNETIC AFFECTION OF LIGHT

BY

MICHAEL FARADAY, F. R. S.

When a ray of polarized light and lines of magnetic force pass simultaneously and parallel to each other through a transparent solid or liquid medium not possessing forces of double refraction, the ray is rotated according to a simple law of action, which I have expressed in the last part of the *Philosophical Transactions.* [*Omitted.*]

Upon consideration it appeared that the peculiar character of the magnetic rotation might be made available in exalting the final effect of the magnetic force upon the ray, and also in demonstrating many important points in a more marked manner and higher degree than had yet been possible; and upon referring the idea to experiment, it was found to be true. The following pages contain some of the results.

A parallelopiped of heavy glass, 0.7 of an inch square and 2.5 inches long, had the two ends polished and silvered. The silvering was then removed from a space about 0.1 of an inch wide along one of the edges of one end, and also from a corresponding space on the other end, except that the parts cleared were on the contrary sides of the parallelopiped; so that each end was furnished with a good plane reflector, but these overlapped each other (Fig. 1). In consequence of this arrangement, a ray of light could be transmitted diagonally across the length of the piece of glass; or the ray, after entering at one end, could be reflected two or more times within the glass and then passed out.

A similar piece of heavy glass was silvered at the two ends and one side of the prism; and the silvering was then removed

at the ends for the space of 0.1 of an inch from those edges which were the farthest from the silvered side (Fig. 2). A ray of light passing in at the unsilvered part of one end with a certain degree of obliquity could be reflected at the other end, then at the side, and again at the first end, passing thus three times along the glass and finally out at the second end. At other inclinations the ray would pass five, seven, nine, eleven, or a greater number of times along the glass before it issued forth on its course through the air to the eye of the observer.

Fig. 1

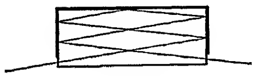

Fig. 2

Either of these pieces of glass could produce the desired result of repeated reflections within, but the first form was found most convenient in use. When a strong light was employed, it was not difficult to follow the series of images produced by successive reflection up to the ninth or tenth image, these corresponding, of course, to a transit of the ray seventeen or nineteen times along the substance of the glass. A little change of position of the silvered glass between the Nicol's prisms used as the polarizing and analyzing apparatus was sufficient to bring any one of these images into view, the glass being at the same time under the full influence of the electromagnet or the helix employed to generate lines of magnetic force. A further advantage is gained if the ends of the piece of glass are not quite parallel to each other, the sides proceeding from the edges where the ray enters and issues forth being in a very slight degree different in length. This arrangement causes the series of reflected images to open out if seen at one end and to close up if seen at the other, and thus the observation of a particular image or the simultaneous comparison of two or more images is favored.

On considering the effect of this arrangement, it is evident that if ABCD represent a trough of solution of sugar, or any other body having the ordinary rotating influence over a polarized ray, then a ray sent in at D and passing out at A would be rotated to a certain amount. But if, instead of proceeding onward at A, it were reflected by the surface AF to E, and

were there observed, it would be found to have received no rotation, for the effect produced in going from D to A would be exactly compensated by its return from A to E. Or, if the reflections were made more numerous and recurred at E, F, and C, so that the ray should traverse the body five times, still an amount of rotation equal only to that which its passage once along the substance could effect would be finally produced.

Such would not be the case if ABCD were a diamagnetic, rotating the ray by means of magnetic force; for then, whichever way the ray was passing, it would still be rotated in the *same* direction in relation to the lines of force. So if observed issuing forth at A, it would have an amount of rotation (which we may call right-handed) equal to what one transit across the diamagnetic could produce; if observed at E, it would have an amount of left-handed rotation double the amount of the first or unit quantity; if observed at F, it would have three times the first amount of right-handed rotation; if observed at C, four times the amount of left-handed rotation; and at B would possess five times the original amount of right-handed rotation.

This was confirmed by the result of an experiment. [*Omitted.*]

Having ascertained the great advantage which this form of apparatus possessed for the examination of many substances which would give no sensible results by the process I formerly described, I proceeded to apply it to the cases of air and some doubly refracting bodies (*Experimental Researches*, 2237). For this purpose I made the faces of the magnetic poles reflectors, by applying to each a polished plate of steel; and as the poles were movable, their reflecting surfaces could be placed at any distance and in any position required, the substance experimented on being between them.

Air.—I could obtain no signs of action upon the ray when air was between the magnetic poles, even with the fourth, fifth, and sixth images.

Rock crystal.—The cubes of this substance, formerly described (*Experimental Researches*, 2178), were submitted to examination; but I could detect no trace of action on the ray of light when passing through them, although they were 0.75 of an inch on the side, and the ray was observed after passing seven and even nine times across them. The cubes were examined in all directions.

Iceland spar.—A cube of this substance (*Experimental Researches,* 2179) was examined in the same manner, but I obtained no effect.

Heavy glass presented the expected phenomena easily and well.

Failing to procure any positive result in these trials, either with air or with doubly refracting crystals, I silvered the latter in the manner that had been employed for the heavy glass, that the magnetic poles might be brought as close as possible; still no evidence of any magnetic action on the ray could be observed.

A natural six-sided prism of rock crystal, 2.3 inches in length, was polished and silvered at the ends; no magnetic effect upon the light could be observed with this crystal with either the first, second, or third image.

M. E. Becquerel thinks that he has observed an effect produced in doubly refracting crystalline bodies, and it is probable that his apparatus is far more delicate for the observance of optical changes than mine. In that case, if combined with the procedure founded on repeated transits of the ray, it perhaps would produce very distinct results; but the latter process alone has not as yet given any evidence of the action sought after.

Certain indications led me to look with interest for any possible effect which the crossing of the reflected rays might produce in the arrangement of the reflectors and glass represented in Fig. 2; but I could find no difference of action between it and the other arrangement, Fig. 1, in which no such crossing occurred.

ROYAL INSTITUTION, *August* 11, 1846.

MICHAEL FARADAY was born September 22, 1791, at Newington, near London, and died August 25, 1867, at Hampton Court. He wrote of himself: "My education was of the most ordinary description, consisting of little more than the rudiments of reading, writing, and arithmetic, at a common day school." In 1804 he became an errand boy, and in 1805 an apprentice in a stationer and bookbinder's shop. Some of the books that passed through his hands aroused an interest in

scientific subjects, and he repeated many of the simpler experiments described in them. In 1810 and 1811 he attended a course of lectures on natural philosophy, and in 1812 heard some lectures on chemistry by Sir Humphry Davy. The latter appointed him an assistant in the Royal Institution in 1813. In 1824 he became a member of the Royal Society, and in 1825 a member of the Royal Institution and director of its laboratory. With the exception of a few years of ill-health, his life until 1862 was one of continuous activity and discovery. For some years longer he carried on administrative and advisory work, but in 1865 he withdrew entirely from active life.

His earlier work was on chemical subjects. In 1831 he began his long series of Electrical Researches. His most important contributions to science were:

1. The discovery of electromagnetic rotation (1821).
2. The liquefaction of gases (1823 and 1844).
3. The discovery of electromagnetic induction (1831).
4. The investigation of the phenomena of electrolysis (1833).
5. The discovery of self-induced electric currents (1834).
6. The discovery of rotation of the plane of polarization of light by substances in a magnetic field (1845).
7. The investigation of diamagnetic phenomena (1845).

His last experiment, made in 1862, was an attempt to discover an effect of magnetism on spectra—the effect which was found by Zeeman in 1897.

The value of his work was widely recognized. He was a member of the most important scientific societies of Europe and America, and received ninety-five honorary titles and orders of merit.

Not only did he contribute more than any other one man to our experimental knowledge of electric and magnetic phenomena, but his speculations as to the important part played by the ether in these phenomena have been fully substantiated by all later investigations.

ON ROTATION OF THE PLANE OF POLARIZATION BY REFLECTION FROM THE POLE OF A MAGNET

BY

JOHN KERR, LL.D.

Mathematical Lecturer of the Free Church Training College, Glasgow

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(*Philosophical Magazine*, Series 5, vol. iii., p. 321, 1877)

ON ROTATION OF THE PLANE OF POLARIZATION BY REFLECTION FROM THE POLE OF A MAGNET

BY

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1. I WAS led some time ago to think it very likely that if a beam of plane-polarized light were reflected under proper conditions from the surface of intensely magnetized iron it would have its plane of polarization turned through a sensible angle in the process or fact of reflection. The known facts upon which this expectation was founded are indicated briefly under the five following heads:

(1) The effects discovered by Faraday in his famous polariscopic experiments in the magnetic field.

(2) Many instances in optics to this effect—that a reflected vibration may have its character determined wholly or partly by the refractive power of the reflector, or, more generally, by the specific properties of the reflecting body in relation to transmitted light. I may adduce Brewster's law of the polarizing angle, also Fresnel's theory of reflection from glass, etc., a theory which is still accepted and applied in delicate photometric work as affording a good expression of facts, and which treats refraction and reflection as closely related parts of one dynamic whole. I may adduce also the laws of reflection from the surfaces of Iceland spar and other birefringent bodies. It is true that in this last case the catoptric effects are extremely faint in comparison with the dioptric—a fact which is clearly

unfavorable to the proposed case of reflection from iron, as contrasted with the resolved cases of transmission through heavy glass, etc. But I think that the following facts bear with equal or greater force the contrary way.

(3) The enormous differences (in relation to magnetic force) between iron and steel on the one side, and Faraday's transparent diamagnetics on the other.

(4) The effects obtained by Verdet in his application of Faraday's magneto-optic method to the salts of iron. The strongest instance is that of the perchloride. A dense solution of perchloride of iron in wood-spirit gives a rotation of light contrary to, and nearly twice as great as, that given by heavy glass under the same conditions.

(5) The known laws of metallic reflection, particularly the fact that silver, zinc, steel, and other metals are distinguished from each other in a perfectly definite manner as reflectors, each metal having specific relations to the principal component vibrations (perpendicular and parallel to the plane of incidence) with reference both to change of phase and change of amplitude.

The preceding facts were sufficient to suggest a plan of procedure as well as to give me a strong expectation of success.

During the month of August last, in the course of some careful experiments in the direction thus indicated, I obtained several interesting results which appeared conclusive. Soon afterwards I gave a description of the experiments before the British Association. Since that time I have made one or two additional observations, and have got rid of a serious error into which I had fallen in my first view of the facts. In this paper I propose to give an account of all my principal experiments and views upon the subject. And first, for future reference, I shall lay down the sum of the results in one sentence.

THE NEW FACT

2. When plane-polarized light is reflected regularly from either pole of an electromagnet of iron, the plane of polarization is turned through a sensible angle in a direction contrary to the nominal direction of the magnetizing current; so that a true south [*nominal north*] pole of polished iron, acting as a reflector, turns the plane of polarization right-handedly.

APPARATUS AND ARRANGEMENTS

3. *The Magnet.*—This is an upright horseshoe electromagnet, and a very good instrument, I think, of its size. Only one limb of the horseshoe is used at a time, the current being sent through one of the coils and the observations being made on the enclosed core. Each of the cores is a solid cylindrical bolt of soft iron, 10 inches long and 2 inches in diameter, which is, therefore, the diameter of each polar surface. Each of the coils weighs 14 pounds, and the wire makes about 400 turns. The particular coil employed in any case is put into circuit (generally as a double wire of 200 turns) with a small Grove's battery of only six cells; and this is the highest power applied in my experiments. In the circuit is placed also a commutator, which is at my hand, so that, while I watch the polariscope, I have the magnetic state of the core under perfect control.

4. *Polar Surfaces.*—These were originally well planed, and perpendicular to the axes of the cores. For the present purpose they had to be smoothed and brightened by polishing, a process which I found troublesome and excessively tedious, from the refractory nature of the material. The polishing was done with fine dry emery powder, applied by chamois leather to one of the surfaces, and by a rubber of fine silk stuff to the other. Each rubber was backed by a flat and smooth block of iron, which was worked carefully by hand over the end of the core. The last stage of the polishing was similar to the earlier stages, but without new additions of emery. When the process was finished, each polar surface (though not such a speculum as would satisfy an optician) acted as a pretty good plane metallic mirror, its plane perpendicular to the axis of the core. Placed in a room in ordinary daylight, each mirror gave good regular images of all surrounding objects that were in any degree illuminated, and, in a darkened room, the image of a neighboring candle flame was generally very good, brilliant, distinct, and sharply and truly outlined, except towards the rim of the mirror, parts not used in the observations. The surface that had been finished by chamois leather was rather more brilliant than the other, but not so perfectly well planed.

I should say here that, from all that I have seen in these experiments and in some earlier trials, I consider the finest attainable polish very desirable. In my present apparatus, I would prefer a much finer polish to any increase whatever of magnetic power (3).

5. *Placing of the Pieces.*—The electromagnet is placed on a solid table, near the edge, and is inclined with its polar surface towards the light by means of a small block placed under the stand. The source of light is a paraffin-flame, narrow and very brilliant, distant a foot or less from the polar surface. Close to the flame stands the first Nicol. The beam of plane-polarized light so rendered is incident horizontally (at an angle of 60° to 80° to the normal) on the polar surface, and is regularly reflected. On this side of the polar surface, a few inches distant, comes the second Nicol, which is supported on a lateral stand, and so placed that, when I look fairly through it, I see the image of the flame in the iron mirror.

6. *Principal Azimuths of First Nicol.*—As the polariscope is worked here in the usual way, by restoration from the best possible extinction, there are only two positions of the first Nicol which are suitable to start from. The plane of polarization of the light incident upon the iron mirror must be either parallel to the plane of incidence or perpendicular to it, because in every other case the reflected light is elliptically polarized, and, therefore, inextinguishable by the analyzer. I generally make the plane of polarization coincide with the plane of incidence; and I manage this in the first place very approximately by trial. I lay the first Nicol with its principal section sensibly horizontal. Looking through the second Nicol, and watching the image of the flame in the polar mirror, I turn the second Nicol quickly through the position of minimum intensity backwards and forwards, while the first Nicol is turned slowly, also backwards and forwards, until I obtain a minimum-intensity zero.

It is a matter of capital importance in the experiments to have the Nicols placed in this position of pure extinction; and the arrangement is not so easily made as might be supposed. Perhaps it is from imperfection of polish, and perhaps from the very nature and structure of the reflecting metal; but whatever be the reason, the mirror is never perfectly black in the

polariscope; and though the intensity of the illumination is very faint when the Nicols are in exact position, it is still sufficient to embarrass the observer's judgment when he has to decide between pure extinction and impure. The difficulty can be overcome by a simple and regular process, as will be seen immediately. In the meantime I assume that we can obtain a pure initial extinction in the polariscope.

7. *Submagnet.* — I have now mentioned everything that is of any importance in the arrangements, except one condition, without which I have never obtained any optical effect; and that is, an intense concentration of magnetic force upon the iron mirror. For this purpose I employ a block of soft iron, one of several polar pieces belonging to the magnet, 2 inches square and 3 inches long, which has been planed off at one end into a blunt wedge with well-rounded edge. Two splinters of hard wood, which have been thinned and toughened by hammering, are laid upon the sloping polar surface about an inch apart and parallel to the plane of incidence. Holding the wedge in my left hand, I plant it edge downwards upon the splinters, with its rounded edge perpendicular to the plane of incidence, and right above the centre of the mirror. The effect of this arrangement is, that when the circuit of the magnetizing current is closed, there is a very powerful concentration of magnetic force upon the mirror, and particularly on that part of it which is utilized optically in the experiments—on so much of it, namely, as the chink between wedge and core leaves exposed, on one side to the lamp, and on the other side to the observer's eye. The lines of magnetic force are sensibly perpendicular to the reflecting surface. The iron mirror is a true polar surface; and its intensely contrasted states, as north, south, neutral, are perfectly under control through the commutator.

The wedge intercepts a large part of the image of the flame. The pieces are generally so placed that the part left of the image is a strong middle segment, both top and bottom being cut off. The object now watched in the polariscope is a broad streak of light, crossing the chink at right angles from top to bottom, very sharply defined, and perfectly suitable as an object in delicate polariscopic work.

Splinters of different thicknesses are employed in different experiments, and in variations of one experiment. My only

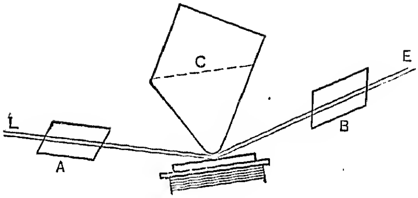


Fig. 1

rule is, that the chink between block and core be as narrow as the requirements of the optical observation will allow. On an average, the width of the chink in the following experiments is about $\frac{1}{20}$ of an inch. The arrangements now described (3–7) are shown simply in the adjacent diagram.

L is the source of light, E the observer's eye, A and B the first and second Nicols, C the wedge of soft iron.

8. *First Experiment.* — The pieces arranged as in the diagram, the chink between block and mirror as narrow as possible, the plane of polarization of the light incident on the polar mirror parallel or perpendicular to the plane of incidence, and the second Nicol turned into the position of pure extinction. The observer now watches the chink through the second Nicol, and works the commutator. When the circuit is closed, the streak of light immediately reappears. The effect is very faint at the best; but it is very distinct and perfectly regular, unless the apparatus is in some way out of order, the mirror dimmed, or the battery working below its average power. Under ordinarily good conditions, at the instant when the circuit is closed the light shows itself faintly in proper form, size, and position across the formerly uniform chink, and so continues without sensible change as long as the current passes. Break, and the light immediately disappears. Reverse, and the light again appears and continues till the instant of break, when it disappears at once.

The beam reflected by the mirror of magnetized iron is certainly not plane-polarized, as is the incident beam (and the reflected beam also before magnetization); for when the light is restored by magnetic force from pure extinction as above, it cannot be extinguished by any rotation of the second Nicol in either direction; nor (as far as I can judge of these faint effects and with the present means) is the light sensibly weakened by any such rotation. The analyzer's position of extinction before magnetization is also (exactly or nearly) the position of minimum intensity after magnetization.

In many repetitions of this experiment, the angle of incidence varied from 60° to 80°, and was generally about 75°.

9. At this point I must ask the reader's attention to several terms, and to the sense in which I shall use them. By a rotation of the first Nicol to the right, I mean a rotation which is right-handed (like the motions of the hands of a watch which faces the observer) when viewed from the point of incidence on the iron mirror. By the north pole of a magnet, I mean "that which points, on the whole, from the north, and, in northern latitudes, upward."*

10. *Second Experiment.*—Taking this experiment as a continuation of the first, and providing for the best effects, I suppose all the arrangements as before: I suppose also that make, break, and reverse of the commutator give bright, black, bright in the polariscope distinctly, however faintly.

(1) Leaving the circuit open and everything else untouched, I simply turn the first Nicol ever so little to the right. The amount of the rotation is important. I have said it was ever so little; and this generally gives effects distinct enough. But when working for the best results I determine the displacement of the first Nicol by this condition, that the intensity of the light restored in the polariscope by the displacement be sensibly equal to that of the light restored formerly by magnetization in the first experiment. This being done, I watch the faint light in the polariscope, and work the commutator as formerly. But I must now specify the magnetic states of the mirror.

When the mirror becomes a north [*south*] pole, the light flashes up at once to a sensibly higher intensity, which is sustained without change as long as the current passes. When the circuit is broken and the mirror demagnetized, the light falls at once from the higher intensity to the primitive faint intensity, and so continues as long as the circuit is open. When

* Sir W. Thomson's papers on *Electrostatics and Magnetism*, § 445. It will be seen from the quotation that this is no innovation of mine. Having had this nomenclature brought to my attention recently by Sir William Thomson, and very strongly recommended by him, I made it a matter of careful consideration, and have determined to adopt it. Like poles of the great earth magnet and of our artificial magnets ought to be similarly named; and the northern pole of the earth magnet cannot with any propriety be called a south pole. [*The contrary definition has been generally adopted.*]

the mirror becomes a south [*north*] pole, the light falls from the primitive faint intensity, down either to perfect extinction or extremely near it. In favorable cases of this kind (that is, in cases properly managed and in a well-darkened room) it is very striking to look at the chink through the analyzer, searching in vain for the faintest trace of the streak of light, and remembering the displacement of the first Nicol. When the circuit is finally broken, the light reappears at once as at first.

(2) Leaving the circuit open and everything else untouched, I watch the faint light in the polariscope, and turn the first Nicol backwards to the left, into the position of extinction and a little beyond it, regulating the amount of rotation by the intensity of the restored light, as in the first case. I now watch the light through the analyzer and work the commutator. It would be superfluous to describe the magnetic changes of the iron mirror, and the corresponding changes in the polariscope; the description would be word for word as before, with one essential alteration. It is the south [*north*] pole that now strengthens the light, and the north [*south*] pole that extinguishes or weakens it.

This experiment is much more easily managed than the first. Let a good sensible extinction of the streak across the chink be obtained by optical trial in the manner already described (6), the plane of polarization of the incident light being either parallel or perpendicular to the plane of incidence; and let the first Nicol be turned to the right so far only as to render the extinction sensibly impure. When the three states of the mirror (north, neutral, south) are now made to succeed one another rapidly, the contrast of bright, faint, dark in the polariscope comes out in almost every case very distinctly.

Very often I have seen the second experiment give clear effects as now described, in cases where, through partial exhaustion of the battery, the first experiment gave no sure effect whatever.

11. I have given these two experiments as a simple and exhaustive summary of a large number of observations which were at first very perplexing, so irregular and apparently inconsistent were the phenomena. The chief cause of my perplexity I found afterwards to be a very interesting thing; and that was what I may truly call the exquisite delicacy of the magnetic mirror as a test for fixing the position of the plane of

polarization of the incident light. One or two simple notes of actual observations will illustrate this point more distinctly than any general statement could do.

Things often happened thus. Working as in the first experiment and with ordinary caution, I started from good extinction, and found the north pole restoring the light and the south pole much the same as open circuit. Trying to obtain better initial conditions if possible, I threw the two Nicols well out of position, and worked them carefully back to good extinction; and now, without any other observable change in the conditions, I found things reversed, the south pole clearly restoring the light and the north pole much the same as open circuit. Here the magnetic mirror simply detected the impurity of the initial extinction, and characterized it, by strong contrasts of intensity in the polariscope, as due to a slight misplacement of the first Nicol (otherwise barely or not at all detectable)—to the right in the first case, and to the left in the second.

Working sometimes with one of the mirrors (that which had been polished by chamois leather, and which was not so well planed as the other) at a particular part of its surface, and at large angles of incidence, I found the upper end of the streak clearly restored by the north pole and the lower end not, while the lower end of the streak was clearly restored by the south pole and the upper end not. There can be no doubt that in this case the magnetic mirror detected a slight difference of slope at those parts of the mirror which reflected the upper and lower ends of the streak. Say that the one part sloped a little downwards to the left, and the other a little downwards to the right; then the planes of incidence at the two places would be out of coincidence with the plane of polarization of the incident light—to the left in the first case, and to the right in the second.

Similarly I have sometimes seen the right side of the streak restored by the north pole and the left side not, while the left side was restored by the south pole and the right side not. Irregularities and inconsistencies of this kind were explained perfectly by the second experiment as soon as it was discovered.

Finally, I observe here that the arrangements for the first experiment are best obtained through those for the second;

and this is a point of some practical importance. Arranging the apparatus as for the first experiment and with the greatest care, I find the effects of the two magnetizations unequal in almost every case. Say that the north pole restores distinctly, and the south pole weakly or not at all. Leaving the circuit open, I turn the first Nicol to the left as little as possible, and then bring the second Nicol into the position of extinction, and test by working the commutator and watching the light in the polariscope. Several careful operations of this kind are sometimes requisite.

SUMMARY AND INTERPRETATION OF THE FACTS

12. In these experiments light is reflected from an iron mirror at an incidence of 60° to 80°, passing through a first Nicol before reflection and through a second Nicol after.

Initial Conditions.—The iron mirror unmagnetized, the principal sections of the two Nicols perpendicular and parallel respectively to the plane of incidence.

Essential Operations.—Starting thus from pure extinction in the polariscope, we apply any one or two of four operations. Two of these are merely mechanical, extremely small rotations of the first Nicol from its initial position, a right-handed rotation (R) and a left-handed (L). The other two are physical, intense magnetizations of the mirror, as a north pole (N) and as a south pole (S).* These four operations will be named here and afterwards by suggestive and easily remembered letters as above; and they will be grouped in pairs invariably, R and N together, thus:

(R, N), (L, S).

13. When any one of the operations is applied singly, the light is restored from pure initial extinction in the polariscope.

When any two of the operations are applied simultaneously, and their relations determined by comparison of effects in the polariscope, they are found to be conspiring operations if they belong to the same pair, and contrary operations if they belong to different pairs. Considering, then, any one of the operations, we see that there are two ways of strengthening its

* [*N and S should everywhere be interchanged, to conform to present usage.*]

effect in the polariscope, and two ways of weakening it. To strengthen the effect of R, apply either operation of the pair (R,N); turn the first Nicol a little more to the right, or magnetize the mirror as a north pole. To neutralize or weaken the effect of R, apply either operation of the pair (L, S); turn the first Nicol a little to the left, or magnetize the mirror as a south pole.

To obtain a complete interpretation of the facts, we have only to assume that the immediate optical effects of the four operations, (R,N), (L,S), are similar in kind for all, and similarly directed for those of either pair, but oppositely directed for different pairs. R and L turn the plane of polarization; so, therefore, according to this view, do N and S. R and N turn the plane of polarization in one direction; L and S turn it in the contrary direction. But even from an optical point of view there is still an important difference between the mechanical operations and the physical; for in the one case (R or L) the full effect of the operation is impressed upon the light before incidence, while in the other case (N or S) the effect is impressed somewhere and somehow in the very process of reflection.

To get a more definite statement of this interpretation, consider the pair of conspiring operations (R,N). In the case of operation N, and to an eye which looks into the polar mirror, the nominal direction of the magnetizing current round the core is right-handed (9). In the case of operation R and to the same eye, the direction of rotation of the plane of polarization, or the direction of rotation of the trace of that plane upon the reflecting surface, is evidently left-handed (9). We infer that a right-handed current gives a left-handed rotation of the plane of polarization. And this completes the first experimental proof of the general statement made in Art. 2.

14. To test the truth of this view of the facts, I thought of three methods which appeared accessible: First, to apply each of the four operations (R,N), (L, S), and to characterize them separately by definite compensating actions in the polariscope; secondly, to apply the operations N and S in combination with small permanent rotations of the second Nicol; thirdly, to return to the case of perpendicular incidence, which I had already tried roughly without success. I shall prepare the way for an account of the first of these methods by a short mathematical discussion.

COMPENSATION OF EFFECTS OF OPERATION R

15. Let the angle of incidence be about 75°, and suppose that the initial conditions are as in the first and second experiments (12), and particularly that the direction OX of the vibration is perpendicular to the plane of incidence. The operation R being now applied, and the incident vibration being turned thus through a small angle, $XOC = \alpha$, it is required to find the character of the reflected light, particularly with a view to compensation. The two rectangular components (one in OX) of the incident vibration (c in OC) are

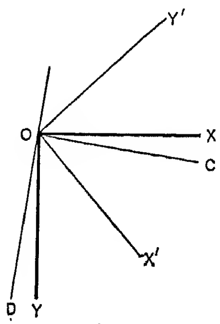


Fig. 2

$$c \cos \alpha \cos 2\pi \frac{t}{\tau},$$

$$c \sin \alpha \cos 2\pi \frac{t}{\tau};$$

or, more briefly,

$$a \cos \theta \text{ and } a' \cos \theta,$$

where θ is proportional to t. Let OY be perpendicular to OX and to the reflected ray; then, to obtain the components x and y of the reflected vibration in the directions OX and OY, we must apply to the preceding components the known laws of metallic reflection. We find thus

$$\left.\begin{aligned} x &= ha \cos \theta \\ y &= ka' \cos (\theta - \phi) \end{aligned}\right\}, \qquad (1)$$

where h and k are constants characteristic of the reflecting metal. As the angle of incidence is about 75°, and, therefore, very near the principal incidence, we may put

$$\phi = \frac{3}{2}\pi. \qquad (2)$$

Substituting in (1), and representing the amplitudes by b and b', we find

$$\left.\begin{aligned} x &= b \cos \theta \\ y &= -b' \sin \theta \end{aligned}\right\}. \qquad (3)$$

From these equations or otherwise we see that the reflected vibration is elliptic, and that its principal rectangular components are perpendicular and parallel, respectively, to the plane of incidence. We see also that the elliptic polarization is left-

handed in the case of operation R and right-handed in the case of L.

Hence a simple method of compensating the effect of the operation R, or of the rotation α of the incident vibration. Introduce a difference of phase $\frac{\pi}{2}$ between the components x and y by means of a quarter-wave plate, and then turn the second Nicol in the proper direction through a small angle which is definitely related to α. This method I have not had an opportunity of trying.

To find another method. Let the elliptic vibration (3) be represented by its rectangular components x' and y', in directions OX′ and OY′ inclined at 45° to OX; and let

$$x'=m\cos(\theta-\beta),$$
$$y'=m\cos(\theta-\gamma).$$

Identifying the second members of these equations with the proper sums of resolved parts of x and y, we find easily

$$m^2=\tfrac{1}{2}(b^2+b'^2),$$
$$\tan\beta=-\frac{b'}{b},\ \tan\gamma=\frac{b'}{b}.$$

And, therefore, if δ be determined by the equation

$$\tan\delta=\frac{b'}{b}=\frac{ka'}{ha}=\frac{k}{h}\tan\alpha, \tag{4}$$

we see that, finally,

$$\left.\begin{aligned} x'&=m\cos(\theta+\delta)\\ y'&=m\cos(\theta-\delta)\end{aligned}\right\}. \tag{5}$$

By any adequate action upon the reflected ray at any point between the iron mirror and the analyzer, let the component x' be retarded relatively to y', so as to undergo a relative change of phase equal to 2δ. As the components x' and y' have already equal amplitudes, and are equally inclined to OX, it is evident that by this change of phase of x' the elliptic vibration (5) is transformed into a rectilinear in the primitive direction OX. And thus the compensation of effect of the operation R is fully effected, without displacement of the second Nicol.

If we assign to $\frac{k}{h}$ the value $\frac{1}{2}$, which is probably near the truth, as its value in the case of steel, measured both by Jamin and by Senarmont, lies between 0.5 and 0.6, and if we give

effect to the condition that α is a very small angle, we find from equation (4), approximately,

$$2\delta=2\tan^{-1}\frac{k}{h}\tan\alpha=2\frac{k}{h}\alpha=\alpha.$$

However, it is sufficient for our present purpose to observe that the compensating change of phase 2δ is a small quantity determined by α, and of the same order as α, and also of the same sign.

16. *The Compensator.*—This is a slip of plate-glass, held in the hands and strained either by flexure round its thickness or by simple tension or compression from the two ends. In the present experiments the slips used were of the best plate, $\frac{1}{8}$ inch thick, $\frac{5}{8}$ wide, and $7\frac{1}{2}$ long, chosen carefully, so as to be quite inactive in the polariscope while unstrained. Suppose one of these slips placed between the mirror and the second Nicol, its surface perpendicular to the reflected ray, and its length parallel to OX′; and let the glass be stretched in the direction of its length. Stretched glass acts upon transmitted light as a positive uniaxal with its axis parallel to the line of extension. In this case, therefore, the extraordinary component x' is retarded relatively to the ordinary y'; and the method found in the last article gives us this simple rule:

To compensate the effect of a small operation R or L, the incident vibration being initially directed along OX, at right angles to the plane of incidence, and the reflected vibration being initially cut off by the second Nicol: Leaving the second Nicol in its initial position, and placing the compensating slip between the mirror and the second Nicol, its plate faces perpendicular to the ray, and its length parallel to OX′, stretch the slip along its length in the case of R, and compress it along its length in the case of L.

The direction OX′ will be taken as the standard direction of strain; it is at 45° to the plane of reflection, right hand down.

17. *Third Experiment.*—All the arrangements are as in the first experiment, the angle of incidence about 75°, and the extinction in the polariscope perfect. As the experiment is a purely optical one, the circuit is kept open. To insure uniformity of optical conditions, the block C is kept in position as in the diagram of (7), and the light is viewed through the chink as formerly.

(1) The first Nicol is turned right-handedly through a very small angle, and the light is distinctly restored. The compensating slip is introduced between the mirror and the second Nicol in the manner which has just been fully described. When the slip is stretched along its length, say between closely gripping finger and thumb at each end, with a force which increases continuously from zero up to a certain small value, the light restored by displacement of the first Nicol fades away to pure extinction, reappearing and brightening as the tension increases. When the slip is submitted to a longitudinal compression which increases continuously from zero, the light increases continuously and very distinctly from beginning to end of the increase of compression.

(2) The first Nicol is now turned to the left, through the position of extinction, and the light distinctly restored; and the compensator, kept always in the standard position, is stretched and compressed as formerly. Things are precisely as in the first case, except that the effects of tension and compression are reversed, and, therefore, interchanged. It is now compression that extinguishes the light; tension strengthens it from first to last.

When the angle of rotation of the first Nicol is too large, which it may be while still very small, the neutralization by tension or compression is incomplete, the light fading to a very sensible minimum and then increasing; but the extinction is still perfect when the initial intensities have reached much greater values than those obtained by magnetization in the first experiment.

I found the present experiment a very interesting one, from the simplicity of the means, the purity of effects, and the beautiful distinctness of the contrasts. However, I do not give the experiment here for its own sake. The only use of it is to characterize the effects of R and L in the polariscope; and this work it does perfectly.

18. *Fourth Experiment.*—The angle of incidence about 75°, and all the arrangements and procedure as in the first experiment, with addition of the compensator. As the intensity in the polariscope is very faint at the best, all proper means are adopted for increasing it—the room well darkened, the battery in good order, the surface of the mirror fresh, the chink between wedge and core merely wide enough to give a good object, and the initial extinction sensibly perfect.

When the light is restored from pure extinction by the operation N, and the compensator is placed and strained as in the third experiment, the light is weakened by tension and strengthened by compression, and the weakening by tension proceeds to pure extinction. The effect of the operation S is, on the contrary, weakened to extinction by compression and strengthened from first to last by tension. [*Paragraph omitted.*]

19. *Fifth Experiment.*—This is a repetition of the second experiment with addition of the compensator; it is more easily managed than the fourth, and the results are equally convincing. In the first half of the second experiment as already described (10), the three sets of operations applied successively were

(R,N), R, (R,S),

and the intensities in the polariscope in the three cases, respectively, were bright, faint, dark.

When the effects in the first and second cases are tested by the compensator, exactly as in the third and fourth experiments, they are both compensated to pure extinction by tension, and both strengthened from first to last by compression. And similarly in the second half of the second experiment, the single effect of L and the joint effect of L and S are both strengthened by tension, and both weakened down to sensible extinction by compression.

20. Summary of the results obtained in the last three experiments.

The effects of the operations R and L in the polariscope are compensated respectively by tension and by compression of glass in the standard direction; the effect of N is compensated precisely as that of R, and the effect of S precisely as that of L; the joint effect of R and N is compensated precisely as the separate effects of R and N, and the joint effect of L and S precisely as the separate effects of L and S; and in all these cases the compensation proceeds to sensible extinction.

The four operations (R,N), (L,S) were found in the second experiment to be related to one another, two and two, as conspiring or contrary; they are now seen to be related to one another more generally, and in the same combinations, as like or unlike. With reference to effects in the polariscope the operations R and S are as clearly unlike as are the operations R and L, or the operations N and S; and, on the other hand, and

with reference always to effects in the polariscope, R and N are as clearly like as are any two operations R, or any two operations N. It was assumed, in explanation of the facts brought out in the second experiment, that the optical effects of the four operations (R,N), (L,S) are the same in kind for all, and similarly directed for those of either pair, but oppositely directed for those of different pairs. All the new facts agree with this hypothesis and confirm it.

It has been observed already that the effects of the operations R and L are fully impressed upon the light before incidence, while the effects of N and S are impressed on the process of reflection; but, as far as we can judge from the present experiments (17, 18, 19), and as far as changes of phase of the principal components are concerned, this difference between the mechanical operations and the magnetic has little influence upon the final effect in the polariscope. We may, therefore, assume provisionally that, as far as changes of phase by metallic reflection are concerned, the rotation due to magnetic force is impressed effectively before incidence. We come now to the second method proposed in 14.

21. *Sixth Experiment.*—Angle of incidence about 75°, initial arrangements as in the first experiment, plane of polarization of the incident light sometimes parallel and sometimes perpendicular to the plane of incidence, initial extinction as pure as possible.

(1) Leaving the first Nicol untouched, I turn the second Nicol right-handedly through a very small angle, and, watching the faint light thus restored, I work the commutator as formerly. The operation N strengthens the light, and this effect is distinct and regular. The operation S has sometimes no effect, and sometimes weakens the light, always less distinctly than N strengthens it, and generally less and less distinctly as the rotation of the second Nicol is diminished.

(2) The second Nicol is turned to the left from its initial position through a very small angle. N and S now interchange effects, but otherwise the phenomena are as in the first case.

The effects obtained in repeated and careful trials were, with few exceptions, as I have now described them; but they were neither strong nor so pure as those obtained in the second experiment. The strengthening actions of N in (1) and of S in (2) are evidently what was to be expected; for in (1) the

second Nicol leaves the plane of polarization a little to the left, and N turns that plane a little more to the left. But the whole subject deserves a more particular discussion.

22. To find the intensity of the light which reaches the observer's eye in the sixth experiment.

Suppose the incident vibration directed along OX (figure of art. 15), at right angles to the plane of incidence. When the second Nicol is turned (right-handedly) through a very small (positive) angle $YOD=\epsilon$ the resolved part of the reflected vibration (of amplitude 1) in the direction OD has an amplitude $=-\sin\epsilon$ or $-\epsilon$, and the intensity of the light transmitted to the eye is ϵ^2.

The effect of an additional operation S is to turn the primitive vibration out of the direction OX through a very small (positive) angle ρ, or to add to the primitive vibration in OX a very small vibration, of amplitude $\sin\rho$ or ρ, in a direction preperpendicular to OX. There are, therefore, two vibrations presented now to the second Nicol—one in OX and sensibly of amplitude 1 as before, the other in OY and of amplitude $k'\rho$ or ρ', where k' is a positive number less than 1, an unknown function of the angle of incidence. According to the hypothesis advanced in the end of art. 20, the difference of phases of these components has the same value ϕ as if the component ρ' in OY were due to an operation R or L. The resolved parts of these components in the direction OD of transmission have amplitudes $-\sin\epsilon$ and $\rho'\cos\epsilon$, or $-\epsilon$ and ρ'; the intensity of the transmitted light is therefore equal to

$$\epsilon^2+\rho'^2-2\epsilon\rho'\cos\phi.$$

23. Before discussing this formula, I proceed to apply similar considerations very briefly to the second experiment. Suppose the direction OX of the primitive vibration still perpendicular to the plane of incidence, and that positive angles are still those due to right-handed rotations. If two operations, L and S, be applied simultaneously, the vibration is turned through a small angle α before incidence, and through a small angle ρ in the process of reflection. The amplitudes of the small reflected vibrations thus generated in the direction OY of transmission may be represented by α' and ρ' where α' is the $k\alpha'$ of equations (1) of art. 15, and ρ' is the same as in art. 22. According to the hypothesis stated in art. 20, these vibrations are reflected

in the same phase, and the intensity of the transmitted light is therefore equal to $(\alpha'+\rho')^2$.

To apply this result to the first half of the second experiment. By trial we give to α such a value that sensibly $\alpha'=\rho'$, and then apply successively the three sets of operations

$$(R,N), (R), (R,S).$$

The corresponding intensities in the polariscope are

$$(-\alpha'-\alpha')^2, (-\alpha')^2, (-\alpha'+\alpha')^2,$$

which are as the numbers 4, 1, 0. The actual results, as already specified, were bright, faint, black (10).

24. Returning to the sixth experiment. In discussing the expression

$$\varepsilon^2+\rho'^2-2\varepsilon\rho'\cos\phi,$$

found in art. 22, I shall suppose the rotation of the second Nicol always right-handed, or the angle ε always positive. The amplitude ρ' is positive for S, negative for N. The angle ϕ varies continuously with the angle of incidence, from zero at normal incidence, through $\frac{1}{2}\pi$ at principal incidence (75° or 76°) up to π at grazing incidence. It will be observed that the $\frac{1}{2}\pi$ at principal incidence in the present case is the $\frac{3}{2}\pi$ of equation (2) of art. 15, diminished by the π of reversal due to reflection.

(1) When the value of the angle of incidence is considerably less than 75°, $\cos\phi$ has some positive value c, and the additions made to the primitive intensity ε^2 by the operations N and S are

$$\rho'^2+2\varepsilon\rho'c \text{ and } \rho'^2-2\varepsilon\rho'c.$$

In this case, therefore, the effect of N in the polariscope is always an increase, and always more pronounced than the effect of S.

Let ε' be the value of ε which is determined by the equation

$$\rho'-2\varepsilon c=0.$$

When $\varepsilon=\varepsilon'$, the effect of S in the polariscope is reduced to zero; when $\varepsilon<\varepsilon'$, the effect of S is a small increase; when $\varepsilon>\varepsilon'$, the effect of S is a decrease.

(2) When the value of the angle of incidence is considerably greater than 75°, $\cos\phi$ has some negative value $-e$, and the additions made to the primitive intensity ε^2 by N and S are

$$\rho'^2-2\varepsilon\rho'e \text{ and } \rho'^2+2\varepsilon\rho'e.$$

Here, therefore, contrary to what holds in the first case, the

effect of S is always an increase, and always more pronounced than the effect of N. Here, also, as e increases from zero, the addition made to ε^2 by the weaker magnetic operation passes from positive, through zero, to negative.

(3) In the case of principal incidence, $\cos\phi=0$, and the additions made by N and S to the primitive intensity ε^2 in the polariscope are equal and always positive.

25. *Seventh Experiment*, a repetition of the sixth to test the preceding inferences.

(1) Angle of incidence about 70°. All the effects recovered as predicted, and as already obtained roughly in the sixth experiment. Recovered also perfectly at various incidences from 60° to 75°.

(2) Angle of incidence very large, about 85°. No sensible effect obtained in any case by application of the operations N and S, with the arrangements of the sixth experiment or with those of the second. The reason very probably is that, as the angle of incidence approximates to 90°, the ratio of the amplitudes ρ' and ε becomes excessively small, by diminution of the rotation ρ towards zero.

Angle of incidence about 80°. The effects very faint, but clearly contrary to what was predicted: N strengthens the light as in the first case; S either weakens it or has no effect.

(3) Equal positive effects of N and S in the polariscope were never observed at 75° or any other incidence. The hypothesis advanced in 20 is therefore inexact: the rotation due to magnetic force is not impressed effectively before incidence. Neither is it impressed effectively after reflection (10...19). The difference of phases of the two reflected vibrations, ρ' in OY and 1 in OX, has therefore some value $\lambda\phi$ intermediate between ϕ and 0; and the intensity in the sixth experiment is equal to

$$\varepsilon^3+\rho'^2-2\varepsilon\rho'\cos\lambda\phi.$$

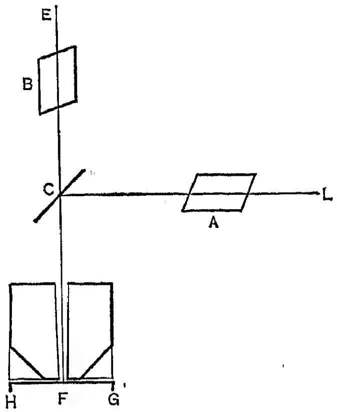
Judging from the earlier experiments, second to fifth, I think we are bound to assume that λ is very nearly eqnal to 1; but certainty upon the subject can be reached only through exact measurements. I come now to the third method mentioned in art. 14.

Case of Perpendicular Incidence.

26. *Submagnet.*—The old wedge C of art. 7 is now inadequate. The piece which I substitute for it is a block of soft

iron, 2 inches square and 3 inches long, rounded at one end into a frustum of a very obtuse cone, of which the small base is hardly $\frac{1}{2}$ inch in diameter. A small boring is drilled through the block, and along the axis of the cone, narrowing regularly from $\frac{1}{4}$ inch at the flat end of the block to $\frac{1}{12}$ inch at the conical end. The surface of the boring is well dimmed with a coating of lampblack. To insure perfect stability of position when the piece rests upon its conical end, the original rectangular volume of the block was restored, the part added being a hard stone of plaster of Paris, which was easily moulded to the block of iron in the usual way. This is the first submagnet that gave me good and constant effects in the case of normal incidence; and it appears to be much the best that I have yet tried. Without a submagnet of some kind, I have never obtained a suspicion of an effect.

27. *Placing of the Pieces.*—The old magnet (3) is placed on a solid table near the edge, with its polar surface horizontal; and the submagnet just described is laid upon one of the polar surfaces, its conical end downwards, the axis of core and boring coincident, and the block and core separated by a wide ring of writing-paper or very thin card. Above the block, as in Norremberg's polariscope, stands a mirror of unsilvered glass, which receives a horizontal beam from the first Nicol, and reflects it downwards through the boring, perpendicularly to the surface of the magnetic mirror. In the diagram, HFG is the polar surface, L the source of light, which is the same paraffin-flame as formerly, E the observer's eye, A and B the first and second Nicols, C the transparent mirror. The course of the light from L to E is LACFCBE. All the pieces are placed very stably, and the room is well darkened.



28. *Eighth Experiment.*—All the pieces are placed as in the diagram, and so that the observer sees at F, through B, a bright and steady image of part of the flame L; the first Nicol is so

laid that the plane of polarization of the light incident at C coincides with the plane of incidence; and the second Nicol is turned into the position of pure extinction.

(1) The second Nicol is turned right-handedly through a small angle, giving a distinct but faint restoration. The operation N strengthens the light thus restored; and the operation S weakens and sometimes extinguishes it.

(2) The second Nicol is turned left-handedly through a small angle beyond pure extinction. The results are as in the first case, with reversal of actions of N and S. It is now S that strengthens the light, and N that weakens or extinguishes it.

The phenomena now mentioned are very faint, a good deal fainter than those obtained in the second experiment; but they are certain, distinct, and perfectly regular. I need hardly say that this experiment is decisive, and that the effects are certainly due to rotations, virtual and actual, of the plane of polarization of the light which is presented to the analyzer, the virtual rotations being produced by displacements of the second Nicol, and the actual by the operations N and S. N conspires with a right-handed rotation of the second Nicol, and therefore N turns the plane of polarization to the left; S conspires with a left-handed rotation of the second Nicol, and therefore S turns the plane of polarization to the right.

29. *Ninth Experiment.*—No change in the arrangements, the initial extinction perfect.

(1) The first Nicol is turned right-handedly (from C as point of view) through a small angle, giving a faint but distinct restoration. S strengthens the light thus restored, and N weakens and sometimes extinguishes it.

(2) The first Nicol is turned left-handedly, through pure extinction to faint restoration. N strengthens the restored light, and S weakens or extinguishes it.

The phenomena are precisely as in the eighth experiment, and equally distinct and regular, but with reversed relations of S and N to movements of the Nicol; and this was to be certainly expected, because the first Nicol simply carries the plane of polarization with it, while the second Nicol simply leaves that plane behind it. [*Experimental proof* (*par.* 30) *omitted.*]

31. *Eleventh Experiment.*—Starting with the same arrangements as in the last three experiments, and working under the most favorable conditions attainable, I have often left the two

Nicols in position at pure extinction, and tried the effects of the simple operations N and S. I have certainly got distinct effects many times in such circumstances, and assured myself that they were due to magnetizations of the iron mirror by getting them to appear and disappear at the instants of make and break of the circuit; but the effects were so excessively faint that I could not once characterize them as due to rotation of the plane of polarization. I have no doubt whatever that, with a stronger magnet and a finer mirror, and a more intense light, this experiment would be as satisfactory as any of the preceding.

32. *Twelfth Experiment: Influence of the Submagnet.* [*Omitted.*]

Summary of Experimental Results.

33. When plane polarized light is reflected perpendicularly from the polar surface of an iron electromagnet, the plane of polarization is turned through a small angle in a direction contrary to the nominal direction of the magnetizing current.

When the light is reflected obliquely, the effect in the polariscope is mixed, partly due to magnetic force, and partly due to metallic reflection; but in this case, as evidently as in the case of normal incidence, the action of the magnetic force is purely or chiefly photogyric, and the plane of polarization is turned always in a direction contrary to that of the magnetizing current.

The precise character of the mixed optical effect in the case of oblique incidence can be determined only by exact measurements. This much, however, appears to be clearly proved by the preceding experiments, that the rotation due to magnetization of the mirror is impressed upon the light neither effectively before incidence nor effectively after reflection.*

No effect was obtained in any case without the presence of a submagnet. I think it certain that the only use of this piece is to concentrate or intensify the magnetic force upon the iron mirror by inductive action.

* [*As in the case of ordinary reflection, the light waves are supposed to penetrate a thin surface stratum of the metal before their direction of motion is reversed. Hence the Kerr effect is closely related to the Faraday effect, and might be predicted from it; but the relation between them is not so simple as might be supposed.*]

The powers applied were barely adequate to produce all the effects. Some of the phenomena were quite imperceptible when the battery began to work, and afterwards, when it had worked at intervals for three or four hours. Much better effects may certainly be expected with higher electromagnetic powers and finer optical appliances.

GLASGOW, 26*th March*, 1877.

ON REFLECTION OF POLARIZED LIGHT FROM THE EQUATORIAL SURFACE OF A MAGNET

BY

JOHN KERR, LL.D.

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CONTENTS

ON REFLECTION OF POLARIZED LIGHT FROM THE EQUATORIAL SURFACE OF A MAGNET

BY

JOHN KERR, LL.D.

In trying to carry forward the magneto-optic inquiry which formed the subject of my last communication to this Magazine,* I proceeded to examine a lateral face of an intensely magnetized iron bar as a reflector, and had the pleasure of obtaining good effects in the first trial. I have lately performed a series of careful experiments on the subject; and I propose to give an account of these and of their very interesting results in the present paper. I mean to describe the experiments at sufficient length for the guidance of any one who would like to repeat them. Most of them are, I think, rather easier and more satisfactory than those described in my former paper.

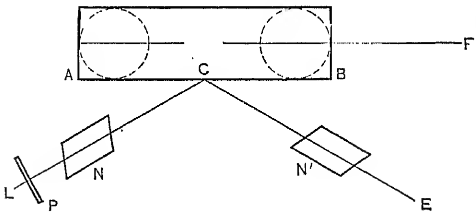
1. *Apparatus.*—The electromagnet is the same upright horseshoe that was used in my former experiments. [*Description omitted.*]

The reflecting bar is a rectangular prism of soft iron, 7 inches long, 2 wide, $\frac{3}{8}$ thick. The iron was selected and specially forged; and its structure is homogeneous and very fine. One lateral face of the bar (7 by $\frac{3}{8}$) was planed and carefully polished by a skilled workman. [*Paragraph omitted.*]

2. *Arrangements.*—The electromagnet stands upright upon a solid table; the reflecting bar lies flat and stably on the poles of the horseshoe, in the position of an armature, its length horizontal and its polished face vertical; the two Nicols and the lamp stand upon the same table as the magnet, and at the same height as the mirror. The diagram shows all the pieces, in

* [*Phil. Mag.*, (5), iii., p. 321, 1877.]

horizontal section through the lamp, L, and the observer's eye, E N is the first Nicol, C the point of incidence on the reflector AB, and N′ the second Nicol. The poles of the horseshoe, below the bar, are indicated by the dotted circles. The piece P between the lamp and the first Nicol was often found useful in the more delicate observations; it is a metallic screen, containing a long horizontal slit about $\frac{1}{8}$ of an inch wide. [*Two paragraphs omitted.*]



It will be observed that, according to these arrangements, the lines of magnetic force at the point of incidence remain sensibly parallel to the trace of the plane of incidence on the reflecting surface, through all changes of incidence from near grazing to near normal.

3. *Specification of Rotations of the Nicols.*—I shall have occasion repeatedly to speak of right-handed and left-handed rotations of the two Nicols. In the employment of these terms, I shall always view the second Nicol from the point E, and the first Nicol from the point C. By a right-handed rotation of the second Nicol I mean, therefore, a rotation of the analyzer which is with watch-hands when viewed from E. . . .

4. *Specification of Magnetizations of the Reflector.*—. . . . By a right-handed current I shall always understand here a magnetizing current whose effective direction, round the reflecting bar AB, is with watch-hands when viewed from the point F.

[*Several paragraphs omitted.*]

6. *First Experiment.*—The plane of polarization of the light incident upon the mirror is constantly parallel to the plane of incidence; and the initial extinction is made as pure as possible.

(1) The second Nicol is turned right-handedly through an extremely small angle from the position of extinction; and the light thus restored faintly in the polariscope is watched for changes of intensity when the reflecting bar is magnetized successively by contrary currents.

(2) The second Nicol is turned left-handedly through an ex-

tremely small angle past the position of extinction; and the optical effects of contrary magnetizing currents are observed as in the former case. The following is an accurate statement of the results :—

(1) The light restored from extinction by a very small displacement of the second Nicol is always strengthened by a right-handed magnetizing current, and always weakened by a left-handed current.

(2) The light restored from extinction by a very small left-handed displacement of the second Nicol is always weakened by a right-handed current and always strengthened by a left-handed current.

The intensity of these optical effects of magnetization varies very noticeably with the angle of incidence. About incidence 85° the effects are very faint, but perfectly regular and much better than merely sensible; about incidence 75° they are more distinct, and very sensibly stronger; about incidences 65° and 60° they are comparatively clear and strong, a good deal stronger than at 75°; about incidence 45° they are still pretty strong, but very sensibly fainter than at 60°; about incidence 30° they are again very faint, much the same as at 85°.

[*Two paragraphs omitted.*]

8. *Second Experiment.*—The plane of polarization of the light incident on the mirror is constantly perpendicular to the plane of incidence. All the other arrangements, and the observations, are precisely as in the first experiment. The second Nicol is turned through an extremely small angle from the position of extinction, first right-handedly, then left-handedly; and in each case the effects of the two magnetizing currents are observed in the polariscope.

About incidence 85°, the light restored by a right-handed rotation of the second Nicol is strengthened by a right-handed current, and so forward, the effects being undistinguishable in any way from those obtained in the first experiment at the same incidence, except that (under equally favorable conditions) they are certainly and considerably fainter. About incidence 80° the effects are still of the same kind, but a good deal fainter—so faint, indeed, that they cannot be brought out very distinctly except under the most favorable conditions (the battery fresh, the initial extinction very pure, and the displacement of the second Nicol extremely small). About incidence 75° the regu-

lar effects disappear. About incidence 70° they reappear very faintly, as faintly as at 80°, but quite distinctly contrary to those obtained at 80° and 85°; the light restored by a right-handed rotation of the second Nicol is now weakened by a right-handed current and strengthened by a left-handed current, and so forward. At incidences 65°, 60°, 45°, 30°, the effects are of the same kind as at 70°, still contrary, therefore, to those obtained at 85°; about incidence 60° they are comparatively clear and strong, though sensibly fainter than those obtained in the first experiment at the same incidence; about 30° they are faint but still distinct, and clearly stronger than the contrary effects obtained at 85°. It appears thus that, in the second experiment, the optical effects of magnetization fall under two distinct cases:

(1) Between grazing and principal incidences, the law is the same as in the first experiment: the right-handed current conspires with a right-handed rotation of the second Nicol, and so forward.

(2) Between principal and normal incidences, this law is simply reversed: the left-handed current conspires with a right-handed rotation of the second Nicol, and so forward. [*Paragraph omitted.*]

10. *Third Experiment.*—The two Nicols are placed initially at pure extinction, the plane of polarization of the light incident on the mirror being parallel to the plane of incidence. The second Nicol remains fixed.

The first Nicol is turned through an extremely small angle from the position of extinction, first right-handedly and then left-handedly; and in each case the effects of the two magnetizing currents are observed in the polariscope.

About incidence 85° the light restored by a right-handed rotation of the first Nicol is weakened by a right-handed current and strengthened by a left-handed current; while the light restored by a left-handed rotation of the first Nicol is strengthened by a right-handed current and weakened by a left-handed current. The effects are fainter than those obtained at the same incidence in the first experiment, but they are distinct and perfectly regular. About incidence 80° the effects are still regular and of the same kind, but very faint, and requiring extremely small displacements of the first Nicol to bring them out clearly. About incidence 75° the

regular effects disappear; and some irregular effects, which make their appearance here, as in the second experiment, are eliminated by the assignment of a proper value to the angle of incidence, and by exact adjustment of the second Nicol.

At incidences 65°, 60°, 45°, 30°, the effects are contrary to those obtained at 85°: the light restored by a right-handed rotation of the first Nicol from extinction is strengthened by a right-handed current, and so forward. At incidence 60°, and even at 45°, the effects are very distinct and comparatively strong, but always fainter than those obtained at the same incidences in the first experiment. It appears thus that in the third experiment, as in the second, the optical effects of magnetization fall under two cases:

(1) Between grazing and principal incidences the right-handed current conspires with a left-handed rotation of the first Nicol, and so forward consistently.

(2) Between principal and normal incidences, the preceding law is simply reversed; the right-handed current conspires with a right-handed rotation of the first Nicol, and so forward.

11. *Fourth Experiment.*—The two Nicols are placed initially at pure extinction, the plane of polarization of the light incident on the mirror being perpendicular to the plane of incidence. All the other arrangements and the procedure are as in the third experiment: the second Nicol remains fixed; and the first Nicol is displaced through a very small angle from the position of extinction, first right-handedly and then left-handedly. The results are very similar to those obtained in the first experiment.

(1) The light restored by a right-handed rotation of the first Nicol is always weakened by a right-handed current, and always strengthened by a left-handed current.

(2) The light restored by a left-handed rotation of the first Nicol is always strengthened by a right-handed current, and always weakened by a left-handed current.

Very near grazing incidence, between 90° and 85°, the effects are insensible; about incidence 85° they are very faint, but regular and quite distinct; they increase in strength quite evidently through the incidences 80°, 75°, 70°, to somewhere between 65° and 60°, where they are, I think, as clear and as strong as those obtained in the first experiment; they then

diminish gradually to incidence 30°, where they are very faint, but still somewhat stronger than at 85°.

It appears thus that, in the fourth experiment, the right-handed current conspires always with a left-handed rotation of the first Nicol.

12. The four experiments which have been described were repeated at several incidences with mirrors of steel. Some finely polished knife-blades were tried, and several masses of other forms. The best was a small bar-magnet of hard steel, which had one of its narrow faces polished on a cutler's wheel (one of the large wheels used for sword-blades). The curvature of this mirror was inconsiderable, and its polish was extremely fine. The arrangements were as in the diagram (2), the bar being laid stably from pole to pole of the horseshoe.

All the old effects were recovered regularly. [*Omitted.*]

13. *Synopsis of the Preceding Results.— Two laws with two exceptions:—*

First Law. The left-handed current conspires with a small right-handed rotation of the analyzer from extinction; and so forward.

Second Law. The right-handed current conspires with a small left-handed rotation of the polarizer from extinction; and so forward.

First Exception. When the plane of polarization of the light incident on the mirror is perpendicular to the plane of incidence, the first law is reversed for all incidences between principal and normal.

Second Exception. When the plane of polarization of the light incident on the mirror is parallel to the plane of incidence, the second law is reversed for all incidences between principal and normal. [*Seven paragraphs omitted.*]

21. I have now given all my positive results; but there are two other lines of experiment which I have tried without effect, and which ought to be briefly noticed.

(1) The mirror, as formerly, an equatorial face of a magnetized bar, the plane of incidence perpendicular to the lines of magnetic force, and the incidence varying from near normal to near grazing. The arrangements were of course somewhat different from those already described; but they were not more difficult, and were made with equal care; and I think that the experiments were altogether as delicate as any of the preceding.

Working in this way at different times, I saw no appearance of optical effect of magnetization.

(2) The mirror an equatorial face of a magnet, the incidence normal, and the inclination of the plane of incidence to the magnetic force varying from 0° to 90°. As the normal incidence was obtained by the employment of a mirror of unsilvered glass, the light was a good deal weaker than formerly; but otherwise the experiment was as delicate as any of the preceding. Nothing like an optical effect of magnetization was observed in any instance.

From these experiments, and from all that I have seen upon the subject, I think it probable, in the highest degree, that magnetization of a reflector even to saturation would be absolutely without optical effect in the cases now exemplified (that is, in the case of normal incidence upon an equatorial face), and in any case where the fronts of the incident and reflected waves are parallel to the lines of magnetic force.

I return now to the consideration of our first arrangement, where the lines of magnetic force are parallel to the intersection of the reflecting surface and the plane of incidence.

22. Law of the optical action of magnetism at incidences near grazing:

Whatever be the angle of incidence between grazing and principal, the effect of magnetization of the mirror, when sensible, is to turn the plane of polarization of the reflected light through a very small angle, in a direction always contrary to that of the Amperian currents; for, whatever be the angle of incidence between grazing and principal, the two laws stated in art. 13 hold true throughout the first four experiments without exception. [*Two paragraphs omitted.*]

Of course this proof assumes, and the conclusion implies, that the reflected light may be considered as approximately plane polarized in all the preceding cases of conspiring actions [*between directions of the current and of the rotations of the Nicols*], as well as in the corresponding instances of mutually compensating actions, both in the optical observations (art. 5) and throughout the first four experiments.

23. It is proved thus beyond question, at least as a very approximate expression of facts, that, near grazing incidence, the effect of magnetization of the mirror upon a reflected ray is to turn the plane of polarization through an extremely small

angle, in a direction contrary to that of the Amperian currents. Under what conditions, on what assumptions, may this be accepted as the law of the action at all incidences? To prepare for a definite answer to this question, I shall first subject the statement of the law to a simple transformation.

When the vibration reflected from the unmagnetized mirror is either parallel or perpendicular to the plane of reflection, the effect of magnetization is to introduce a new and very small component vibration in a direction perpendicular to the primitive vibration, the sense of the new component being that assumed by the primitive vibration when turned through a right angle in a direction contrary to the Amperian currents. And for incidences between grazing and principal, the difference of phases of the two components (the primitive and the new) is much nearer to 0 than to $\frac{1}{2}\pi$.

It is important to observe here that the results obtained in the fifth and sixth experiments* necessitate the assumption of some such law as this, even for incidences between principal and normal. For the direction of the primitive vibration is in those experiments exactly parallel or perpendicular to the plane of reflection, and the Nicols remain constantly in their initial position of pure extinction; so that the observed effects of magnetization, effects which are of the same kind as those produced by rotation of the second Nicol, cannot be explained by any mere changes of the primitive vibration in amplitude or phase, or by anything except the introduction of a new and very small component in a direction perpendicular to the primitive vibration.

24. *General Law of the Action of Magnetism upon the Reflected Ray:*

The three following assumptions appear to me to afford a perfect explanation of all the principal phenomena. They were suggested as above, and were tested by a careful mathematical discussion of the results of all the experiments in succession. The discussion presents little difficulty, but is too tedious to be offered here.

* [*In these experiments the Nicols remained in the positions for extinction, and the effects of magnetization were indicated by a small upward or downward motion of a dark horizontal band extending across the field of view. Similar displacements of this band were caused by very slight rotations of either Nicol.*]

(1) When the original vibration is parallel or perpendicular to the plane of reflection, the effect of magnetization of the mirror is to turn the vibration through a very small angle in a direction contrary to the Amperian currents.

The resolved parts of the vibration so turned, one in the direction of the primitive vibration and the other perpendicular to it, may be called the primitive component and the new component respectively, as in art. 23.

(2) The primitive component is always reflected according to the same laws of retardation of phase, after magnetization of the reflector as before.

(3) Whether the new component be parallel or perpendicular to the plane of reflection, and whatever be the angle of incidence, the phase retardation of the new component (with reference to a standard reflected ray polarized in the plane of incidence, and incident in the same phase as the actual primitive) is always an angle in the first quadrant, and much nearer to zero than to $\frac{1}{2}\pi$.

It will be admitted that the assumption (3) is a very remarkable one, and very important if true. I hope to see this geometric theory of the phenomena verified by the mathematicians, or something better put in its place.

25. It would be superfluous now to offer any explanation of the absence of all optical effects of magnetization in the case of normal incidence (art. 21). It is not so easy to understand the absence of effect at incidences very near grazing, 85° to 90°, in the first four experiments. We might expect, indeed, on the contrary, that as the ray approaches parallelism to the lines of force, or as the front of the wave approaches perpendicularity, the magnetic force would act at a greater advantage, and the optical effect would therefore become stronger. But against this we have what appears to be fairly established by observation as a general truth in optics, that the specific differences of reflectors become less and less marked as incidence approaches grazing, until at grazing they almost entirely disappear. [*Two paragraphs omitted.*]

27. The first facts of magneto-optics discovered long ago by Faraday, their more immediate consequences discovered afterwards by Verdet and others—these and the additional facts now published by myself must all be included ultimately under one physical theory. It is very probable that the remarkable

theory of magnetism which has been advanced by Sir William Thomson* in a discussion of the former class of facts will apply as well to the latter. Probably also the theory itself may receive additional confirmation in the process. I think it equally probable that the new facts will find important applications in the mechanical parts of the wave-theory. But in any event there is a new physical action secured thoroughly to science, a specific action of magnetized iron upon light incident at any point of its surface.

GLASGOW, *January* 21, 1878.

BIOGRAPHICAL SKETCH OF KERR

THE REV. JOHN KERR, LL.D., F.R.S., was born December 17, 1824, at Ardrossan, Scotland. He was a student in Glasgow from 1841 to 1846, and at the Theological College of the Free Church of Scotland, in Edinburgh, in 1849. Since 1857 he has been mathematical lecturer in the Free Church Training College for Teachers, in Glasgow.

His most important experimental work has been the discovery of double refraction in solid and liquid dielectrics in an electrostatic field (1875) and of the effect described in the preceding pages.

* [*Proc. R. S., June*, 1856. *See also Maxwell, Phil. Mag., March, April, and May*, 1861, *and Jan. and Feb.*, 1862.]

ON THE INFLUENCE OF MAGNETISM ON THE NATURE OF THE LIGHT EMITTED BY A SUBSTANCE

BY

DR. P. ZEEMAN

(*Philosophical Magazine*, Series 5, 43, p. 226, 1897*)

* §§ 1–10 were read before the Academy of Sciences at Amsterdam October 31, 1896, and §§ 11–25 November 28, 1896.

CONTENTS

ON THE INFLUENCE OF MAGNETISM ON THE NATURE OF THE LIGHT EMITTED BY A SUBSTANCE

BY

DR. P. ZEEMAN*

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1. SEVERAL years ago, in the course of my measurements concerning the Kerr phenomena, it occurred to me whether the light of a flame if submitted to the action of magnetism would perhaps undergo any change. The train of reasoning by which I attempted to illustrate to myself the possibility of this minor importance at present,† at any rate I was induc to try the experiment. With an extemporized app spectrum of a flame, coloured with sodium, placed poles of a Ruhmkorff electromagnet, was looked at. was negative. Probably I should not have tried t ment again so soon had not my attention been draw years ago to the following quotation from Maxwell's Faraday's life. Here (Maxwell, *Collected Works*, ii. read: "Before we describe this result we may men in 1862 he made the relation between magnetism and lig subject of his very last experimental work. He endeavoured, but in vain, to detect any change in the lines of the spectrum of a flame when the flame was acted on by a powerful magnet." If a Faraday‡ thought of the possibility of the above-mentioned relation, perhaps it might be yet worth while to try the experi-

* Communicated by Prof. Oliver Lodge, F. R. S., with the remark that he had verified the author's results so far as related to emission spectra and their polarization.

† *Cf.* § 15 and § 16.

‡ See Appendix for Faraday's own description of the experiment.

ment again with the excellent auxiliaries of spectroscopy of the present time, as I am not aware that it has been done by others.* I will take the liberty of stating briefly to the readers of the *Philosophical Magazine* the results I have obtained up to the present time.

2. The electromagnet used was one made by Ruhmkorff and of medium size. The magnetizing current furnished by accumulators was in most cases 27 amperes, and could be raised to 35 amperes. The light used was analyzed by a Rowland grating, with a radius of 10 ft., and with 14,438 lines to the inch. The first spectrum was used, and observed with a micrometer eye-piece with a vertical cross-wire. An accurately adjustable slit is placed near the source of light under the influence of magnetism.

3. Between the paraboloidal poles of an electromagnet, the middle part of the flame from a Bunsen burner was placed. A piece of asbestos impregnated with common salt was put in the flame in such a manner that the two D-lines were seen as narrow and sharply defined lines on the dark ground. The distance between the poles was about 7 mm. If the current was put on, the two D-lines were distinctly widened. If the current was cut off they returned to their original position. The appearing and disappearing of the widening were simultaneous with the putting on and off of the current. The experiment could be repeated an indefinite number of times.

4. The flame of the Bunsen was next interchanged with a flame of coal-gas fed with oxygen. In the same manner as in § 3, asbestos soaked with common salt was introduced into the flame. It ascended vertically between the poles. If the current was put on again, the D-lines were widened, becoming perhaps three or four times their former width.

5. With the red lines of lithium, used as carbonate, wholly analogous phenomena were observed.

6. Possibly the observed phenomenon (§§ 3, 4, 5,) will be regarded as nothing of any consequence. One may reason in this manner: widening of the lines of the spectrum of an incandescent vapour is caused by increasing the density of the radiating substance and by increasing the temperature.† Now, under

* See Appendix.

† *Cf.*, however, also Pringsheim (*Wied. Ann.*, xlv., p. 457, 1892).

the influence of the magnet, the outline of the flame is undoubtedly changed (as is easily seen), hence the temperature, and possibly also the density of the vapour, is changed. Hence one might be inclined to account in this manner for the phenomenon.

7. Another experiment is not so easily explained. A tube of porcelain, glazed inside and outside, is placed horizontally between the poles, with its axis perpendicular to the line joining the poles. The inner diameter of the tube is 18 mm., the outer one 22 mm. The length of the tube is 15 cm. Caps are screwed on at each end of the tube ;* these caps are closed by plates of parallel glass at one end, and are surrounded by little water-jackets. In this manner, by means of a current of water, the copper caps and the glass plates may be kept sufficiently cool while the porcelain tube is rendered incandescent. In the neighbourhood of the glass plates, side-tubes provided with taps are fastened to the copper caps. With a large Bunsen burner the tube could be made incandescent over a length of 8 cm. The light of an electric lamp, placed sideways at about two meters from the electromagnet, in order to avoid disturbing action on the arc, was made to pass through the tube by means of a metallic mirror. The spectrum of the arc was formed by means of the grating. With the eye-piece the D-lines are focused. This may be done very accurately, as in the centre of the bright D-lines the narrow reversed lines are often seen. Now a piece of sodium was introduced into the tube. The Bunsen flame is ignited and the temperature begins to rise. A coloured vapour soon begins to fill the tube, being at first of a violet, then of a blue and green colour, and at last quite invisible to the naked eye. The absorption soon diminishes as the temperature is increased. The absorption is especially great in the neighbourhood of the D-lines. At last the two dark D-lines are visible. At this moment the poles of the electromagnet are pushed close to the tube, their distance now being about 24 mm. The absorption-lines now are rather sharp over the greater part of their length. At the top they are thicker, where the spectrum of the lower, denser vapours was observed. Immediately after the closing of the current the lines *widen* and are seemingly *blacker ;* if the current is cut

* Pringsheim uses similar tubes in his investigation concerning the radiation of gases, *l. c.* p. 430.

off they immediately recover their initial sharpness. The experiment could be repeated several times, till all the sodium had disappeared. The disappearance of the sodium is chiefly to be attributed to the chemical action between it and the glazing of the tube. For further experiments, therefore, unglazed tubes were used.

8. One may perhaps try to account for the last experiment (§ 7) in this way: It is true that the tube used was not of the same temperature at the top and at the bottom; further, it appears from the shape of the D-lines (§ 7) that the density of the vapour of sodium is different at different heights. Hence certainly convection-currents caused by difference of temperature between the top and bottom were present. Under certain plausible suppositions one may calculate that, by the putting on of the electromagnet, differences of pressure are originated in the tube of the same order of magnitude as those caused by the difference of temperature. Hence the magnetization will push, *e. g.*, the denser layer at the bottom in the direction of the axis of the tube. The lines become widened; for their width at a given height is chiefly determined by the number of incandescent particles at that height in the direction of the axis of the tube. Although this explanation still leaves some difficulties, certainly something may be said for it.

9. The explanation of widening of the lines attempted in § 8 is no longer applicable to the following variation of the experiment, in which an unglazed tube was used. The inner diameter of the tube, about 1 mm. thick, was 10 mm. The poles of the electromagnet could be moved till the distance was 14 mm. The tube was now heated by means of the blowpipe instead of with the Bunsen burner, and became in the middle part white hot. The blowpipe and the smaller diameter of the tube made it easier to bring the upper and lower parts to the same temperature. This is now higher than before (§ 7), and the sodium lines remain visible continuously.* One now can wait till the density of the sodium vapour is the same at various heights. By rotating the tube continuously round its axis I have still further promoted this. The absorption-lines now are equally broad from the top to the bottom. When the electromagnet was put on, the absorption-lines immediately

* Pringsheim, *l. c.* p. 456.

widened along their whole length. Now the explanatio.. in the manner of § 8 fails.

10. I should like to have studied the influenee of magnetism on the speetrum of a solid. Oxide of sodium has, as was found by Bunsen or Bahr, the remarkable property of giving by incandescence a spectrum with bright lines. With the dispersion used, however, the edges of these lines were too indistinct to serve my purpose.

11. The different experiments from §§ 3 to 9 make it more and more probable that the absorption and hence also the emission lines of an incandescent vapour are widened by the action of magnetism. Now if this is really the ease, then, by the action of magnetism on the free vibrations of the atoms, whieh are the cause of the line-speetrum, other vibrations of changed period must be superposed. That it is really inevitable to admit this specific action of magnetism is proved, I think, by the rest of the present paper.

12. From the representation I had formed to myself of the nature of the forces acting in the magnetic field on the atoms, it seemed to me to follow that with a band-spectrum and with external magnetic forces the phenomena I had found with a line-spectrum would not occur.

It is very probable that the difference between a band and a line spectrum is not of a quantitative but of a qualitative kind.* In the case of a band-spectrum the molecules are complicated; in the case of a line-spectrum the widely separated molecules contain but a few atoms. Further investigation has shown that the representation I had formed of the cause of the widening in the case of a line-spectrum in the main was really true.

13. A glass tube, closed at both ends by glass plates with parallel faces and containing a piece of iodine, was plaeed between the poles of the Ruhmkorff eleetromagnet in the same manner as the tube of porcelain in § 7. A small flame under the tube vaporized the iodine, the violet vapour filling the tube.

By means of eleetrie light the absorption-spectrum could be examined. As the temperature is low this is the band-speetrum. With the high dispersion used, there are seen in the bands a very great number of fine dark lines. If the eurrent

* Kayser in Winkelmann's *Handbuch*, ii., 1, p. 421.

round the magnet is closed, *no* change in the dark lines is observed, which is contrary to the result of the experiments with sodium vapour.

The absence of the phenomenon in this case supports the explanation, that even in the first experiment, with sodium vapour (§ 7), the convection-currents had no influence. For in the case now considered the convection-currents originated by magnetism, which I believed to be possible in that case, apparently are insufficient to cause a change of the spectrum; yet, though I could not see it in the appearance of the absorption-lines (*cf.* § 7), the band-spectrum is, like the line-spectrum, very sensible to changes of density and of temperature.

14. Although the means at my disposal did not enable me to execute more than a preliminary approximate measurement, I yet thought it of importance to determine approximately the value of the magnetic change of the period.

The widening of the sodium lines to both sides amounted to about $\frac{1}{40}$ of the distance between the said lines, the intensity of the magnetic field being about 10^4 C. G. S. units. Hence follows a positive and negative magnetic change of $\frac{1}{40000}$ of the period.

15. The train of reasoning mentioned in (1), by which I was induced to search after an influence of magnetism, was at first the following: If the hypothesis is true that in a magnetic field a rotatory motion of the ether is going on, the axis of rotation being in the direction of the magnetic forces (Kelvin and Maxwell), and if the radiation of light may be imagined as caused by the motion of the atoms, relative to the centre of mass of the molecule, revolving in all kinds of orbits, suppose for simplicity circles; then the period, or, what comes to the same, the time of describing the circumference of these circles, will be determined by the forces acting between the atoms, and then deviations of the period to both sides will occur through the influence of the perturbing forces between ether and atoms. The sign of the deviation, of course, will be determined by the direction of motion, as seen from along the lines of force. The direction will be the greater the nearer the plane of the circle approximates to a position perpendicular to the lines of force.

16. Somewhat later I elucidated the subject by representing to myself the influence exercised on the period of a vibrating system, if this is linked together with another in rapid rotatory

motion. Lord Kelvin (now 40 years ago)* gave the solution of the following problem: Let the two ends of a cord of any length be attached to two points at the ends of a horizontal arm made to rotate round a vertical axis through its middle point at a constant angular velocity, and let a second cord bearing a material point be attached to the middle of the first cord. The motion now is investigated in the case where the point is infinitely little disturbed from its position of equilibrium. With great angular velocity the solution becomes rather simple. Circular vibrations of the point in contrary directions have slightly different periods. If for the double pendulum we substitute a luminiferous atom, and for the rotating arm the rotational motion about the magnetic lines of force, the relation of the mechanical problem to our case will be clear.

It need not be proved that the above-mentioned considerations are at most of any value as indications of somewhat analogous cases. I communicate them, however, because they were the first motive of my experiments.

17. A real explanation of the magnetic change of the period seemed to me to follow from Prof. Lorentz's theory.†

In this theory it is assumed that in all bodies small electrically charged particles with a definite mass are present, that all electric phenomena are dependent upon the configuration and motion of these "ions," and that light-vibrations are vibrations of these ions. Then the charge, configuration, and motion of the ions completely determine the state of the ether. The said ion, moving in a magnetic field, experiences mechanical forces of the kind above mentioned, and these must explain the variation of the period. Prof. Lorentz, to whom I communicated these considerations, at once kindly informed me of the manner in which, according to his theory, the motion of an ion in a magnetic field is to be calculated, and pointed out to me that, if the explanation following from his theory be true, the edges of the lines of the spectrum ought to be circularly polarized. The amount of widening might then be used to determine the ratio between charge and mass, to be attributed in this theory to a particle giving out the vibrations of light.

* *Proc. Roy. Soc.*, 1856.

† Lorentz, *La Théorie électromagnétique de Maxwell.* Leyden, 1892; and *Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern.* Leyden, 1895.

The above-mentioned extremely remarkable conclusion of Prof. Lorentz relating to the state of polarization in the magnetically widened lines I have found to be fully confirmed by experiment (§ 20).

18. We shall now proceed to establish the equations of motion of a vibrating ion, when it is moving in the plane of (x, y) in a uniform magnetic field in which the magnetic force is everywhere parallel to the axis of z and equal to H. The axes are chosen so that if x is drawn to the east, y to the north, z is upwards. Let e be the charge (in electromagnetic measure) of the positively charged ion, m its mass. The equations of relative motion then are:

$$\left.\begin{aligned} m\frac{d^2x}{dt^2} &= -k^2x + e\mathrm{H}\frac{dy}{dt} \\ m\frac{d^2y}{dt^2} &= -k^2y - e\mathrm{H}\frac{dx}{dt} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)^*$$

The first term of the second member expresses the elastic force drawing back the ion to its position of equilibrium; the second term gives the mechanical force due to the magnetic field. They are satisfied by

$$\left.\begin{aligned} x &= \alpha e^{st} \\ y &= \beta e^{st}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

provided that

$$\left.\begin{aligned} ms^2\alpha &= -k^2\alpha + e\mathrm{H}s\beta \\ ms^2\beta &= -k^2\beta - e\mathrm{H}s\alpha \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

when m, k, e are to be regarded as known quantities.

For us the period T is particularly interesting. If $\mathrm{H}=0$, it follows from (3) that

$$s = i\frac{k}{\sqrt{m}} = i\frac{2\pi}{\mathrm{T}},$$

or

$$\mathrm{T} = \frac{2\pi\sqrt{m}}{k}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If H is not 0, it follows from (3) approximately that

$$s = i\frac{k}{\sqrt{m}}\left(1 \mp \frac{e\mathrm{H}}{2k\sqrt{m}}\right).$$

* These equations are like those of the Foucault pendulum, and of course lead to similar results.

Putting T′ for the period in this case, we have

$$\mathrm{T}' = \frac{2\pi\sqrt{m}}{k}\left(1 \pm \frac{e\mathrm{H}}{2k\sqrt{m}}\right). \quad . \quad . \quad . \quad . \quad (5)$$

Hence the ratio of the change of period to the original period becomes

$$\frac{e\mathrm{H}}{2k\sqrt{m}} = \frac{e}{m} \cdot \frac{\mathrm{HT}}{4\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

A particular solution of (1) is that representing the motion of the ions in circles. If revolving in the positive direction (*viz.*, in the direction of the hands of a watch for an observer standing at the side towards which the lines of force are running), the period is somewhat less than if revolving in the negative direction. The period in the first case is determined by the value of (5) with the minus sign, in the second with the plus.

The general solution of (1) shows that the ions describe, besides circles, also slowly rotating elliptical orbits. In the general case, the original motion of the ion having an arbitrary position in space, it is perfectly clear that the projection of the motion in the plane of (x, y) has the same character. The motion resolved in the direction of the axis of z is a simple harmonic motion, independent of and not disturbing the one in the plane of (x, y), and hence one not influenced by the magnetic forces. Of course, the consideration of the motion of an ion now given is only to be regarded as the very first sketch of the theory of luminiferous motions.

19. Imagine an observer looking at a flame placed in a magnetic field in a direction such that the lines of force run towards or from him.

Let us suppose that the said observer could see the very ions of § 18 as they are revolving; then the following will be remarked: There are some ions moving in circles and hence emitting circularly polarized light; if the motion is round in the positive direction the period will, for instance, be longer than with no magnetic field; if in the negative direction, shorter. There will also be ions seemingly stationary and really moving parallel to the lines of force with unaltered period. In the third place, there are ions which seem to move in rotating elliptical orbits.

If one desires to know the state of the ether originated by the moving ions, one may use the following rule, deduced by

Prof. Lorentz from the general theory: Let us suppose that in a molecule an ion P—of which the position of equilibrium is P_0—has two or more motions *at the same time, viz.*, let the vector P_0P always be obtained by adding the vectors P_0P which should occur in each of the component motions at that moment; then the state in the ether at a very great distance in comparison with P_0P will be obtained by superposing the states which would occur in the two cases taken separately.

Hence it follows in the first place that a circular motion of an ion gives circularly polarized light to points on the axis of the circle.

Further, we may choose instead of the above-considered elliptical orbits a revolution more suited to our purpose. One may resolve the motion of the ion, existing before the putting on of the magnetic force, into a rectilinear harmonic motion parallel to the axis of z and two circular (right-handed and left-handed) motions in the plane of (x, y).

The first remains unchanged under the influence of the magnetic force, the periods of the last are changed.

By the action of the grating the vibrations originated by the motion of the ions are sorted according to the period, and hence the complete motion is broken up into three groups. The line will be a triplet. At any rate, one may expect that the line of the spectrum will be wider than in the absence of the magnetic field, and that the edges will give out circularly-polarized light.*

20. A confirmation of the last conclusion may be certainly taken as a confirmation of the guiding idea of Prof. Lorentz's theory. To decide this point by experiment, the electromagnet of § 2, but now with pierced poles, was placed so that the axes of the holes were in the same straight line with the centre of the grating. The sodium lines were observed with an eye-piece with a vertical cross-wire. Between the grating and the eye-piece were placed the quarter-undulation plate and Nicol which I formerly used in my investigation of the light normally reflected from a polarly magnetized iron mirror.†

* I saw afterwards that Stoney, *Trans. Roy. Soc. Dublin*, iv., endeavours to explain the existence of doublets and triplets in a spectrum by the rotation of the elliptical orbits of the "electrons" under the influence of perturbing forces.

† Zeeman, *Communications of the Leyden Laboratory*, No. 15.

The plate and the Nicol were placed relatively in such a manner that right-handed circularly polarized light was quenched. Now, according to the preceding, the widened line must at one edge be right-handedly circularly polarized, at the other edge left-handedly. By a rotation of the analyzer over 90° the light that was first extinguished will be transmitted, and *vice versa*. Or, if at first the right edge of the line is visible in the apparatus, a reversal of the direction of the current makes the left edge visible. The cross-wire of the eye-piece was set in the bright line. At the reversal of the current the visible line moved! This experiment could be repeated any number of times.

21. A small variation of the preceding experiment is the following. With unchanged position of the quarter-wave plate the analyzer is turned round. The widened line is then, during one revolution, twice wide and twice fine.

22. The electromagnet was turned 90° in a horizontal plane from the position of § 20, the lines of force now being perpendicular to the line joining the slit with the grating. The edges of the widened line now appeared to be plane-polarized, at least in so far as the present apparatus permitted to see, the plane of polarization being perpendicular to the spectral line. This phenomenon is at once evident from the consideration in § 19. The circular orbits of the ions, being perpendicular to the lines of force, are now seen on their edges.

23. The experiments in §§ 20 to 22 may be regarded as a proof that the light-vibrations are caused by the motion of ions, as introduced by Prof. Lorentz in his theory of electricity. From the measured widening (§ 14) by means of relation (6), the ratio e/m may now be deduced. It thus appears that e/m is of the order of magnitude 10^7 electromagnetic C.G.S. units. Of course this result is only to be considered as a first approximation.

24. It may be deduced from the experiment of § 20 whether the positive or the negative ion revolves.

If the lines of force were running towards the grating, the right-handedly circularly polarized rays appeared to have the greater period. Hence in connection with § 18 it follows that the negative * ions revolve, or at least describe the greater orbit.

* In the original paper it was incorrectly stated that the positive ions revolve. I pointed out this mistake in *Phil. Mag.* for July, 1897. [*Note added by author.*]

25. Now that the magnetization of the lines of a spectrum can be interpreted in the light of the theory of Prof. Lorentz, the further consideration of it becomes specially attractive. A series of further questions already present themselves. It seems very promising to investigate the motions of the ions for various substances, under varying circumstances of temperature and pressure, with varying intensities of the magnetization. Further inquiry must also decide as to how far the strong magnetic forces existing according to some at the surface of the sun may change its spectrum.

The experiments described have been made in the Physical Laboratory at Leyden, to the Director of which, Prof. Kamerlingh-Onnes, I am under great obligations for continuous interest in the present subject.

AMSTERDAM, *January*, 1897.

APPENDIX

SINCE the publication of my original paper in the *Proceedings of the Academy at Amsterdam*, and while the present paper was in the press, I have become acquainted with two attempts, till now unknown to me, in the same direction, and also with the original account of Faraday's experiment referred to in § 1. The last is to be found in Faraday's *Life*, by Dr. Bence Jones, vol. ii., p. 449 (1870), and as it is extremely remarkable I will reprint it here:

"1862 was the last year of experimental research. Steinheil's apparatus for producing the spectrum of different substances gave a new method by which the action of magnetic poles upon light could be tried. In January he made himself familiar with the apparatus, and then he tried the action of the great magnet on the spectrum of chloride of sodium, chloride of barium, chloride of strontium, and chloride of lithium."

On March 12 he writes: "Apparatus as on last day (January 28), but only ten pairs of voltaic battery for the electromagnet.

"The colorless gas-flame ascended between the poles of the magnet, and the salts of sodium, lithium, etc., were used to give color. A Nicol's polarizer was placed just before the intense magnetic field, and an analyzer at the other extreme of the apparatus. Then the electromagnet was made, and unmade, but not the slightest trace of effect on or change in the lines in the spectrum was observed in any position of polarizer or analyzer.

"Two other pierced poles were adjusted at the magnet, the colored flame established between them, and only that ray taken up by the optic apparatus which came to it along the axis of the poles, *i. e.*, in the magnetic axis, or line of magnetic force. Then the electromagnet was excited and rendered neutral, but not the slightest effect on the polarized or unpolarized ray was observed.

"This was the last experimental research that Faraday made."

In 1875 we have a paper by Prof. Tait, who has kindly sent me a copy, "On a Possible Influence of Magnetism on the Absorption of Light, and some correlated subjects" (*Proc. Roy. Soc. of Edinburgh*, Session 1875–76, p. 118). Prof. Tait remarks that a paper by Prof. Forbes, read at the Society, and some remarks upon it by Maxwell, have recalled to him an experiment tried by him several times, but which hitherto has led to no result. Then the paper proceeds:

"The idea is briefly this. The explanation of Faraday's rotation of the plane of polarization of light by a transparent diamagnetic requires, as shown by Thomson, molecular rotation of the luminiferous medium. The plane-polarized ray is broken up, while in the medium, into its circularly polarized components, one of which rotates with the ether so as to have its period accelerated, the other against it in a retarded period. Now, suppose the medium to absorb one definite wave-length only, then—if the absorption is not interfered with by the magnetic action—the portion absorbed in one ray will be of a shorter, in the other of a longer, period than if there had been no magnetic force; and thus, what was originally a single dark absorption-line might become a double line, the components being less dark than the single one."

Hence here the idea is perfectly clearly expressed of the experiment, tried in vain; an idea closely akin to that of § 15 above, both being in fact founded on Kelvin's theory of the molecular rotation of the luminiferous medium, though not directly applicable to the experiment of § 9, in which case the lines of magnetic force are perpendicular to the axis of the tube.

In the second place I have to mention two papers by the late M. Fievez, to which attention has been drawn by M. van Aubel, in a letter to Prof. Onnes and intended for communication to the Academy of Sciences, Amsterdam. Prof. Onnes read the letter at the January meeting, and made at the same time some explanatory remarks of which in the following I make free and extensive use. The papers referred to are: M. Fievez, "De l'Influence du Magnétisme sur les caractères de Raies spectrales" (*Bulletin de l'Acad. des Sciences de Belgique*, 3^{e} série, tome ix., p. 381, 1885); and Fievez, "Essai sur l'Origine des Raies de Fraunhofer, en rapport avec la Constitution du Soleil" (*l. c.* 3^{e} série, tome xii., p. 30, 1886). Here experiments are

described as in §§ 4 and 13 of the present paper. Nothing, however, is observed about the widening of the absorption-lines, nor about the polarization of the emitted light. The results obtained by M. Fievez merit careful attention and consideration. He has observed with a flame in a magnetic field not only widening, but reversal and double reversal of the lines of the spectrum, the lines at the same time becoming more brilliant. Unfortunately, quantitative details are not given. The facts observed in some cases by Fievez are qualitatively not in accordance with my observations or what is to be deduced from my results. Hence even in the cases where the results are qualitatively in accordance, the question remains whether Fievez has observed *the same phenomenon.* The field used by Fievez seems to have been more intense than the one I had at my disposal. Is it possible perhaps to account in this manner for the "double renversement (c'est-à-dire l'apparition d'une raie brillante au milieu de la raie noire élargie)"? I think the answer must be in the negative. For, arguing from § 19, a line must widen, or else, the field being very intense, become a triplet. We cannot but understand from Fievez's description of the experiment that the light was emitted perpendicularly to the lines of force. Now the double reversed line of Fievez is not the triplet to be expected from theory, for it is expressly stated by Fiévez that the line experimented upon is not the simple line of the spectrum, but one previously widened and reversed (by some agency independent of magnetism). By the action of magnetism a brilliant line in the centre of the black line appears. Hence, perhaps, one may interpret the case of double reversal as a direct action of magnetism, but then only as a doubling of the absorption-line, and not as a division of the original line into three parts. As the application of Lorentz's theory given in § 18 is confessedly only a very first sketch, further theoretical and experimental evidence is wanted before we are able to decide whether in the experiment of Fievez a specific action of magnetism on light or an effect of perturbing circumstances was observed. Indeed, one may make the same objection to M. Fievez's experiment as I myself have made to my own analogous experiment in § 6.

The whole of the phenomena observed by Fievez can readily be attributed to a change of temperature by the well-known actions of the field upon the flame (change in its direction or

outline, magnetic convection, etc.); and the last sentence of his paper states that "les phénomènes qui se manifestent sous l'action du magnétisme sont identiquement les mêmes que ceux produits par une élévation de température." The negative result obtained by Fievez with absorption-spectra would without further consideration (as in § 12) point in the same direction. The inference to be drawn from Fievez's experiments alone would rather be, I think, that the temperature of the flame is changed in his experiments than that a specific action of magnetism on the emission and absorption of light exists. By experiments already in progress I hope to settle the dubious points.

Summarizing, we may say: Had the experiments of Fievez come to my knowledge they would have been a motive for me to further investigation, Fievez not having prosecuted his inquiry up to a decisive result. At least, at present it remains even doubtful whether the phenomenon observed by Fievez with a magnetized flame is really to be attributed to *the specific action of the magnetic field on the period of the vibrations of light*, which I have found and undoubtedly proved by the experimental confirmation of Lorentz's predictions.

AMSTERDAM, *February*, 1897.

DOUBLETS AND TRIPLETS IN THE SPECTRUM PRODUCED BY EXTERNAL MAGNETIC FORCES

BY

DR. P. ZEEMAN

(I. *Philosophical Magazine*, Series 5, 43, p. 55, 1897.)
(II. *Philosophical Magazine*, Series 5, 43, p. 255, 1897.)

CONTENTS

PART I

PART II

DOUBLETS AND TRIPLETS IN THE SPECTRUM PRODUCED BY EXTERNAL MAGNETIC FORCES

BY

DR. P. ZEEMAN

PART I

1. ON a former occasion * I have remarked that the elementary treatment of the general idea of the Lorentz theory of the magnetic broadening of the spectral-lines indicates that the broadened line must in some cases be broken up into a triplet. I have examined this subject somewhat more in detail. Further consideration shows that with a very strong magnetic field, a magnetically broadened line must be broken up into doublets or triplets according as the light is emitted in a direction parallel or perpendicular respectively to the lines of force. With a smaller intensity of the field the line will be simply widened. The prediction of Lorentz with regard to the polarization of the edges of the broadened lines having been fulfilled, it seemed worth while to pursue still further the study of the polarization of the lines. Even with magnetic forces insufficient to break up the line into a triplet, it might be possible to learn something more about the constitution of the widened line. It seemed to me to be of interest to investigate this point, and to see whether the explanation formerly given, intended as the very first sketch of the motion of ions according to the theory of Lorentz, was further confirmed by experiment.

* *Phil. Mag.* for March, 1897.

Prof. Oliver Lodge * has suggested that, under some suppositions, we might conclude also that the line will be simply widened in a strong field. I take the liberty of stating briefly to the readers of the *Philosophical Magazine* the preliminary results I have obtained in regard to these questions.

2. In § 18 of the cited paper, the motion of an ion in a magnetic field was treated according to Lorentz's theory. The system of co-ordinates was chosen in such a manner that the axis of z was parallel to the magnetic force, the plane of (x, y) being perpendicular to it. Referring for the rest to the cited paper, I recapitulate that the considered motions of the ions, existing before the putting on of the magnetic force, were resolved into a rectilinear harmonic motion parallel to the axis of z and two circular (right-handed and left-handed) motions in the plane of (x, y).

The first remains unchanged under the influence of the magnetic force, the periods of the last are changed.

3. Using the rule given by Lorentz, therefore, we see that in the direction of the lines of force, right-handedly and left-handedly circularly polarized light of changed period must be propagated. The ions vibrating parallel to the lines of force of course do not cause any disturbance in the ether equivalent to light in that direction.

In a direction perpendicular to the lines of force, those motions of the ions which have a component parallel to the plane of (x, y) give plane-polarized light vibrating in a vertical plane and with changed period (I suppose the axis of z in a horizontal plane). The ions, however, moving parallel to the axis of z give electrical vibrations in a horizontal direction with unchanged period.

4. *Magnetic Doublets.*—Hence with every value of the magnetic force, an infinitely narrow spectral-line breaks up into two lines, the light being emitted in the direction of the lines of force. However, for lines of finite width the magnetic change of the period must exceed somewhat that corresponding to *half the width* of the original line in order that the doublet may be seen. One of the components then must be over the whole width left-handedly, the other right-handedly circularly polarized.

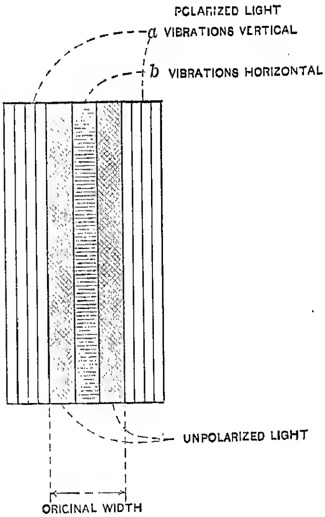
* *The Electrician* for February 28, 1897, p. 569.

5. *Magnetic Triplets.*—With finite width of the spectral-line and observing in a direction perpendicular to the lines of force, the line is broken up into a triplet if the magnetic change of the period corresponds to *the whole width* of the original spectral-line. Hence the magnetic force necessary to produce a triplet is equal to twice the value necessary for a doublet.

6. *Intermediate Forms of Magnetic Doublets and of Triplets.*—With magnetic forces less than the ones supposed in §§ 4 and 5, forms intermediate between the unchanged spectral-lines and the doublets and triplets may be expected. With the doublet only one particular intermediate form exists, viz., a line the edges of which are circularly polarized, the central part emitting unpolarized light. This case I described in my former paper.

Of triplets two intermediate forms may be distinguished.

I will call a line "triplet *a*" if the magnetic change corresponds to somewhat more than half the width of the unchanged line. In this case the broadened line is composed in the following manner: the central part will emit horizontal vibrations, at both its sides bands of (chiefly, but not entirely) unpolarized light border it, which again are enclosed by bands of vertically vibrating light. The accompanying diagram gives a rough scheme of the constitution of the line.



In "Triplet *b*" the magnetic change amounts to less than half the width of the original line. This case I described in my cited paper.

7. Hence we may expect that if we can just resolve the doublet, the magnetic force is also sufficient for triplet *a*.

As with sodium (*cf.* § 11) I did not succeed completely with the means at my disposal in observing the expected doublets and triplets, so with other

substances I looked in the first place for magnetic doublets. We might expect, then, that further inquiry would show something about the triplets. At last I succeeded in observing with the blue line ($\lambda=480\ \mu\mu$) of cadmium the doublet and the triplet *a*. It has already been remarked by Egoroff and Georgiewsky * that cadmium also exhibits the broadening of the spectral-lines under the action of magnetism.

8. The method of experiment was principally the same as that formerly used. As in my former experiments, I used a Ruhmkorff electromagnet. I missed, however, the beautiful Rowland grating I used in the laboratory of Prof. Onnes. I now had at my disposal only a smaller one with a radius of 6 ft., but like the Leyden one with 14,438 lines to the inch. The second spectrum was very satisfactory. For the cadmium spectrum a spark was used between cadmium electrodes; with a lens, as often used in this manner by Lockyer, an image of spark and electrodes was formed on the slit; it is then easily verified that the part of the spark just between the poles is analyzed.

The above-mentioned cadmium-line is especially sharp at the violet side.

Now I succeeded, indeed, in observing the expected phenomena (doublet and triplet *a*).

9. For observing the doublet along the lines of force a perforated pole was used. With a current of 30 amp., the distance between the poles being as small as possible without interfering with the cadmium electrodes, a perfectly defined doublet was seen. One of the components appeared to be left-handedly, the other right-handedly circularly polarized † *over its whole width.* It might still be argued that the dark space between the components is caused by a reversal, and that it is not a doubling of the lines. However, it must then be supposed that this absorption-line just intercepts that part of a magnetically

* *C. R.*, 1897, t. cxxiv., pp. 242, 748.

† The sign of the circular polarization is the same in the case of this cadmium-line and of the D-lines. I must, however, correct my statement in § 24 of my former paper. I now see that if the lines of force are running towards the grating, the right-handedly circularly polarized rays appear to have the greater period. Hence the radiation is due chiefly to the motion of a *negatively*-charged particle. Probably my mistake arose from a faulty indication of the axis of the $\frac{1}{4}\lambda$ plate used.

broadened line which emits polarized light. Now with a smaller value of the magnetic force the dark space becomes narrower. I grant that this may be the case also with the supposed absorption-line. If so, a greater part of the broadened emission-line should emit unpolarized light, the smaller absorption-line now intercepting but a part of it. It appeared, however, that even with a very narrow intervening dark space, the components of the doublet were entirely circularly polarized, and hence the intervening dark space seems to be different from an absorption-line.

There is yet another argument against the interpretation that the dark space is of the nature of an absorption-band. In that case one should expect also to see reversal if looking across the lines of force. However, in this direction no trace of an ordinary absorption-line was visible with the same magnetic intensity. Hence we must conclude, I think, that I have really observed the doublet indicated by Lorentz's theory.

10. Looking across the lines of force, I succeeded in observing the phenomenon indicated as triplet a (§ 6, diagram). If no Nicol was used, only a broadened line was visible. A Nicol oriented so as to get rid of light whose plane of polarization is vertical makes a dark, well-defined line appear in the midst of the broadened line. Turning the Nicol 90°, this dark line disappears, the lateral wings now being quenched, because these are polarized in a horizontal plane. All this is to be expected according to § 6. The analyzers in §§ 9 and 10 were inserted, according to Lodge's* advice, before the light reached the grating.

Until now I have not had the opportunity of measuring the intensities of the fields used in §§ 9 and 10. I took care, however, to make them nearly equal in the two cases.

11. Finally it may be mentioned that I observed also traces of the phenomena described in §§ 9 and 10 with sodium. As the D-lines reverse so very easily by non-magnetic agency, and as they show sometimes complicated phenomena (Lockyer and Ciamician),† it seems difficult to see the phenomena pure and simple with sodium. It is true that it is not difficult to avoid perturbations caused by the motion of the piece of asbestos, as

* *The Electrician*, 26 February, 1897.

† *Cf.* Kayser, *Spectralanalyse*, p. 305.

mentioned by Egoroff and Georgiewsky * in their interesting note. To avoid disturbances of this kind I simply moistened the mouth-piece of the burner with a very concentrated solution of NaCl or NaBr. Small disturbances, nevertheless, easily make their appearance.

12. Although my inquiry is not yet closed, I think, however, that we can say that it seems to give new evidence in favour of the interpretation of the magnetization of the spectral-lines given by Lorentz's theory. When I have determined the intensity of my field, we can decide the question whether we can hope to obtain a pure triplet by augmentation of the magnetic force or by using narrower spectral-lines, though, of course, it may yet turn out that the phenomena will prove to be less simple. For a future paper must be reserved also the accurate measurement of the amount of the magnetic change with cadmium and other substances, † and therewith the discussion concerning the ratio between mass and charge of the ions in Lorentz's theory. It is very probable that these "ions" differ from the electrolytical. It is true that by means of the latter many phenomena can be interpreted, as also is done in a paper by Richarz, ‡ where, *e. g.*, the molecular magnetism is explained by the motion of the ions, but the high value of e/m which I have found makes it extremely improbable that we have to deal with the same mass in the two cases. Lodge § has pointed out in an extremely interesting paper that my value of e/m can be explained even without the necessary motion of any "matter" at all.

Appendix.—On communicating the observations of this paper to Prof. Lodge, he most kindly informed me that he had already seen some of them himself. Especially he had seen the doubling of the lines and shown them at the Royal Society soirée on May 20. He, however, did not suppose it to be a real doubling of the line, but thought it to be a broadening and a reversal. I have given my reasons (the chief being the circular polarization over the whole width) for thinking that I observed

* *C. R.*, t. cxxiv., p. 949.

† I have experiments in progress (measurements of the photographed magnetized spectrum) by which I hope to be able to obtain fairly accurate values of the magnetic change.

‡ *Wied. Ann.*, lii., p. 385 (1894).

§ *The Electrician*, March 12, 1897.

a real doubling. I am confirmed in this opinion because it corroborates the observation of the perpendicular polarization of the middle and of the edges of the triplet *a* (§ 10). As Prof. Lodge does not mention that he observed this last phenomenon, and as my point of view is different from his, his object not being in the first place to test Lorentz's theory, I publish the paper in its original form, though it turns out that part of its contents has been observed by both of us independently.*

AMSTERDAM, *June* 4.

* *Disclaimer by* Prof. LODGE.

Having been asked to exhibit Prof. Zeeman's discovery at the Royal Society soirée, I arranged apparatus to obtain it more powerfully, and thereby saw the new effects, but without any intention of trespassing on the prerogative of the discoverer.

What I saw concerning polarization was that, when looking across the lines of force, a Nicol brought out the doubling (or tripling or quadrupling, as the case might be) more sharply, no doubt by quenching the residual light; and that it restored the original appearance of the line when rotated through 90°. I mention this here as partly confirming Prof. Zeeman's most interesting and much more complete investigation, on the theoretical aspect of which at present I offer no further opinion.—LIVERPOOL, *June* 6.

PART II.*

13. *Magnetic Triplet.*—With the grating of § 8 and with a very strong current and rightly shaped conical poles I succeeded in observing also the pure triplet (§ 5) with cadmium. Again the blue cadmium-line was examined which I have used for the other characteristic phenomena. The field used was about 32,000 C.G.S. Using this strong field and looking without a Nicol across the lines of force the cadmium-line was seen tripled, *i. e.*, broken up into *three* lines, separated by dark spaces. The existence of this triplet demonstrates, as it seems to me, irrefragably, and independently of the examination of the state of polarization, the magnetic nature of the phenomenon.

If, now, a Nicol is placed in the rays with its plane of polarization horizontal, then only the two outer lines of the triplet are seen. A rotation of the Nicol through 90° makes the central line appear and quenches completely the light of the outer ones. Hence the central line of the triplet emits plane-polarized light, the plane of polarization being vertical; the outer lines, on the other hand, emit light polarized in a horizontal plane. This result entirely confirms the considerations of §§ 3 and 5. The question of § 12 is now answered. New evidence in favour of the interpretation by Lorentz's theory of the magnetization of the spectral-lines has been obtained.†

14. *Measurement of Magnetic Change.*—The triplet and also the "triplet *a*" of § 6 enable us to measure accurately the magnitude of the magnetic change; on a former occasion‡ I have given only the result of a rough measurement in order to determine the order of magnitude. Using the "triplet *a*," I have now obtained a far more reliable value. Looking across the lines of force and quenching the horizontal vibrations by means of a Nicol, the vertical vibrations only emerge. Using a

* The §§ 13–17 were communicated to the June meeting of the Amsterdam Academy, § 18 is now (July, 1897) added.

† The line 4678 becomes a triplet, 4800 a quadruplex.—(Author's note of January, 1900.)

‡ *Phil. Mag.* March, 1897, p. 230.

grating, there are to be seen two separate lines, consisting of vertical vibrations. The distance between the centres of these lines corresponds to the double change of the period. Of course, this distance can be measured far more accurately than the widening of a line. The accuracy of the measurement by means of a micrometer eye-piece is much increased if the grating gives brilliant lines. This quality is possessed by a grating in the possession of the laboratory of the University of Groningen. Its Director, Prof. Haga, kindly invited me to make some measurements with his apparatus, which was in full working order. The grating is mounted in a very stable manner, which, of course, is very favourable for accurate measurements.

15. For the particulars of the mounting I refer to Haga's paper (*Wied. Ann.*, lvii., p. 389, 1896). The grating (best quality) has a radius of 10 ft. and 10,000 lines to the inch. The source of light used was a piece of asbestos paper soaked with molten salt and introduced into the flame of coal-gas fed with oxygen under high pressure. An image of the sodium flame was formed on the slit by means of a lens. Between the lens and the slit a large Nicol was placed; the distance between the slit and the flame was about 50 cm. Care was taken that there were no absorption-lines in the spectrum of the non-magnetized flame.

16. The Nicol was placed so that its plane of polarization was horizontal. With the putting on of the current the two lines mentioned in § 14 appear (*cf.* also § 10). The distance between these lines was now measured by means of a micrometer eye-piece. The movable frame carries cross wires; the cross resembles that of St. Andrew. For spectroscopic measurements this cross is recommended (see, *e. g.*, Scheiner, *Spectralanalyse der Gestirne*, p. 74). Illumination of the wires was necessary. Now the position of each of the four lines formed by the D-lines was read. The difference between two readings gives the distance between the centres of D_1 and D_2 in terms of the divisions of the screw-head (one revolution = 100 divisions). These differences are entered in the following table. The electromagnet soon became very hot by the heat generated by the necessary current (22 amp.) and by the action of the flame. Hence it was impossible to make more than three or four measurements without interrupting the current.

Thirty-eight measurements give for the distance between D_1

and D_2 288 divisions. The probable error of one measurement of the magnetic change is 6.5 divisions for D_1, 4.5 for D_2. The results have the probable errors 1.5 and 1.0 divisions. The magnetic change is the same for the two sodium-lines, the difference lying within the limits assigned by the probable errors. The intensity of the field (determined by a bismuth spiral) was 22,400 C.G.S. In this field the positive and negative magnetic change of the period amounts to $\frac{1}{17,800}$. Hence e/m is 1.6×10^7.

Distances between the Centres in divisions of screw-head

for D_1	for D_2
26	36
18	30
26	32
45	37
25	46
28	36
38	46
42	26
26	33
33	25
35	32
53	28
36	31
51	21
26	34
26	35
25	37
31	25
21	25
Mean 32.3 ± 1.5	32.4 ± 1.0

On a former occasion I have found by a rough measurement for the magnetic change $\frac{1}{40,000}$, the field being 10,000 C.G.S. Assuming proportionality between change and intensity, the present measurements give for this field a change of $\frac{1}{39,800}$. The close agreement between the result of my rough measurement and the new one is of course the effect of mere chance, for the magnetic change in the case of sodium depends on temperature,*

* *Phil. Mag.*, March, 1897, p. 227, §§ 3 and 4.

which may have been different in the two cases.* Probably it was not very high in the case now considered. The order of magnitude of e/m is entirely the same as the one formerly given.

17. The great brightness of the Groningen grating makes it possible to observe very clearly with sodium also the "triplet a," which I described for cadmium. If no Nicol is used the exciting of the magnet seems to break up the sodium-line into two lines, the phenomenon somewhat resembling the one observed if a Nicol is used, but with this difference, that the darker part is not very dark and not narrowed; hence the appearance differs considerably from the one we are accustomed to observe with reversals.

The explanation may be (as was remarked by Prof. Haga to the author) that now, the three constituents of the triplet partially overlapping one another, the maxima are conspicuous and the inner part appears dark by contrast. Thus there would be no reversal in our case. This really seems to be so, as is confirmed by the following experiment. The above-mentioned hazy dark line being visible, a Nicol was placed in the beam so as to get rid of light whose plane of polarization is vertical; now only the bright line which emits horizontal vibrations remains visible, but without the slightest trace of a reversal. For this observation it is of course very desirable to use a bright grating. I had no opportunity of obtaining a measurement of the doublet to be seen along the lines of force.

18. A few words may perhaps be said here concerning Prof. Michelson's paper in the July number of the *Philosophical Magazine*. Michelson applies his beautiful method of the interferometer to our subject, and there can certainly be but one opinion as to its particular advantages in such cases. Some of his results seem at first sight at variance with mine. Only in one case is there perfect agreement between Michelson's results and mine, viz., when the light is emitted along the lines of force. We both get a doublet in this case. Michelson adds that a broadening is inappreciable. Evidently he means a broadening of the components, which I did not advocate, though it exists in a slight degree in some cases. I referred in my first paper only

* Recent measurements give $\frac{1}{25,000}$ for the magnetic change of D_1 in a field of 10,000. For D_2 the value is somewhat smaller.

Vide Cotton, *Phénomène de Zeeman*, p. 60.—(Author's note of January, 1900.)

to a broadening, because I had not yet resolved the doublet or the triplet. It is of course a proof of the superiority of Michelson's method that with relatively weak magnetic forces he could recognize the duplicity, whereas for me it was not so easy.

Looking across the lines of force I have observed a triplet, whereas Michelson also for this case finds a doublet, though less clearly marked than in the other case. This certainly at first sight seems a great difference. I venture, however, to give the following explanation:

Let us assume, not only that the visibility curve is practically the same as that due to a doublet, but also that it is undoubtedly proved that a triplet cannot give a practically identical curve in some cases, of which I for myself am not sure at present. Granting that we must assent to a doublet, the question arises whether there cannot be assigned another cause for the difference. I think there can. Michelson in making his experiments was yet unacquainted with the particular states of polarization in the triplet. I think that a sufficient reason for the difference mentioned may be found in the perpendicular polarization of the outer lines and the central one of the triplet (§ 13) or of the central band and the outer edges of triplet *a* (§ 6).

Supposing that the apparatus was arranged in a horizontal plane, then the reflections (under 45°) from the two plane plates, one unsilvered and one semi-silvered with a transparent film of silver, must weaken the horizontal vibrations especially and hence almost annihilate their influence. For unsilvered plates it is easily calculated that the intensity of horizontal vibrations is, under the circumstances stated, weakened five or six times more than that of the vertical ones. This ratio is of course somewhat changed by the silver film. How much, cannot be said without knowledge of further details.

If I have indicated the real cause, Michelson has observed a case analogous to my triplet *a*, when a Nicol is interposed in such a position as to get rid of the horizontal vibrations. Let us hope that Prof. Michelson will soon give us his opinion.

AMSTERDAM, *July* 10.

[*Note added August 10.*]—Prof. Michelson has kindly informed me that he believes my explanation of the discrepancy in our results to be correct.

Biographical Sketch of Zeeman

Pieter Zeeman was born May 25, 1865, at Zonnemaire, near Zieriksee, Netherlands. In 1885 he entered the University of Leyden, where he became assistant in the physical laboratory in 1890. In 1893 he worked at the University of Strasburg, and in the same year received his doctor's degree from the University of Leyden. In 1894 he was appointed privat-docent in that university, in 1897 he became lecturer on experimental physics in the University of Amsterdam, and he is now (1900) professor of experimental physics in that university.

Zeeman's inaugural dissertation was on "Measurements Concerning the Kerr Effect," and he has since done much valuable work on that subject. Altogether he has published more than twenty papers since 1893, most of them dealing with the relations between magnetism and light. He has also investigated other optical phenomena and the propagation and absorption of electric waves in liquids.

In 1898 he became a member of the Royal Academy of Sciences of Amsterdam. He is, further, on the list of various learned societies.

In 1899 the Imperial Academy of Sciences at Vienna awarded him the Baumgartner Prize, and the Académie des Sciences at Paris the prix Wilde.

BIBLIOGRAPHY

Faraday Effect

General

Wiedemann. *Pogg. Ann.*, 82, p. 215, 1851.

Verdet. *Ann. Chim. Phys.* (3), 41, p. 370, 1854; 43, p. 375, 185; 69, p. 415, 1863.
Comptes Rendus, 39, 1854; 43, 1856; 44, 1857; 56, 1863.
Œuvres de Verdet, tom. 1, p. 112, 1872.

H. Becquerel. *Ann. Chim. Phys.* (5), 12, p. 5, 1877; (5), 27, p. 312, 1882.
Jour. de Phys. (1), 8, p. 198, 1879; 9, p. 265, 1880.
Comptes Rendus, 86, 1878; 89, 1880; 90, 1880.

Sohncke. *Wied. Ann.*, 27, p. 203, 1886.

In Gases

Kundt and Röntgen. *Wied. Ann.*, 6, p. 332, 1879; 8, p. 278, 1879; 10, p. 257, 1880.

H. Becquerel. *Comptes Rendus*, 88, 1879.

Siertsema. *Verh. Akad. Wetensch.*, Amsterdam, 1894, 1899.
Jour. de Phys. (3), 288, 1898.
Arch. Néerl. (2), 2, p. 261, 1898.

In Magnetic Metals

Kundt. *Wied. Ann.*, 23, p. 228, 1884; 27, p. 191, 1886.
Phil. Mag. (5), 18, p. 308, 1884.

du Bois. *Wied. Ann.*, 31, p. 941, 1887; 35, p. 137, 1888.

Lobach. *Wied. Ann.*, 39, p. 347, 1890.

In Liquids and Solutions

Perkin. *Jour. Chem. Soc.*, since 1884.

Jahn. *Wied. Ann.*, 43, p. 280, 1891.

Ostwald. *Jour. Chem. Soc.*, 59, p. 198, 1891.

Wachsmuth. *Wied. Ann*, 44, p. 377, 1891.

Schönrock. *Zeit. phys. Chem.*, 11, p. 753, 1893.

Humburg. *Zeit. phys. Chem.*, 12, p. 401, 1893.

In Doubly Refracting Bodies

E. Becquerel. *Ann. Chim. Phys.* (3), 17, p. 445, 1846.

Lüdtge. *Pogg. Ann.*, 137, p. 271, 1869.

Wedding.	*Wied. Ann.*. 35, p. 25, 1888.
Chauvin.	*Comptes Rendus*, 102, 1886; 108, 1889.
Gouy.	*Jour. de Phys.* (2), 4, p. 149, 1885.

KERR EFFECT

Hall.	*Phil. Mag.* (5), 12, p. 157, 1881.
Kundt.	*Phil. Mag* (5), 18, p. 308, 1884.
	Wied. Ann., 23, p. 228, 1884.
Righi.	*Ann. Chim. Phys.* (6), 4, p. 433, 1885; 9, p. 65, 1886; 10, p. 200, 1887.
du Bois.	*Phil. Mag.* (5), 29, p. 253, 1890.
	Wied. Ann., 39, p. 25, 1890.
Sissingh.	*Wied. Ann.*, 42, p. 115, 1891.
	Arch. Néerl. 27, p. 173, 1893.
Zeeman.	*Arch. Néerl.* 27, p. 252, 1893; 27, p. 274, 1894; (2) 1, p. 220, 1897; 1, p. 376, 1898.

THEORIES OF FARADAY AND KERR EFFECTS

Rowland.	*Phil. Mag.* (5), 11, p. 254, 1881.
H. A. Lorentz.	*Arch. Néerl.* 19, p. 123, 1884.
Potier.	*Comptes Rendus*, 108, 1889.
Bassett.	*Proc. Royal Society*, 49, p. 76, 1891.
	Trans. Royal Society, 182 (A), p. 371, 1892.
Drude.	*Wied. Ann.*, 46, p. 353, 1892; 48, p. 122, 1893; 49, p. 690, 1893; 52, p. 496, 1894; 62, p. 687, 1897.
Goldhammer.	*Wied. Ann.*, 46, p. 71, 1892; 47, p. 345, 1892; 48, p. 740, 1893; 50, p. 772, 1893; 65, p. 111, 1898.
Rieff.	*Wied. Ann.*, 57, p. 281. 1896.
Larmor.	*Phil. Mag.* (5), 42, p. 201, 1896.
	British Assoc. Rept., 1893 (Review of Theories).
Wind.	*Verhand. Phys. Ges. Berlin*, 13, p. 84, 1895.
	Arch. Néerl. (2), 1, p. 119, 1897; 1, p. 369, 1899.
	Physical Review, 6, p. 43, 1897; 7, p. 98, 1898.

ZEEMAN EFFECT

General

Lodge.	*Proc. Royal. Soc.*, 60, p. 513. 1897; 61, p. 413, 1897.
Michelson.	*Astrophysical Journal*, 6, p. 48, 1897; 7, p. 131, 1898.
	Phil. Mag. (5), 45, p. 348, 1898.
Cornu.	*Jour. de Phys.* (3), 6, p. 673, 1897.
	Comptes Rendus, 125, 1897; 126, 1898.
Becquerel.	*Jour. de Phys.* (3), 6, p. 681, 1897.
	Comptes Rendus, 125, 1897; 126, 1898; 127, 1898.

Becquerel and Deslandres. *Comptes Rendus*, 126, 1897; 127, 1898.
Egoroff and Georgiewsky. *Comptes Rendus*, 124, 1897; 125, 1897.

Righi.	*Act. R. Accad. dei Lincei* (5), 7, p. 295, 1898.
	Comptes Rendus, July, 1898.
	Il Nuovo Cimento, Ser. 4, t. 10, p. 20, 1899.

Ames, Earhart, and Reese. *Johns Hopkins University Circulars*, 17, p. 53, 1898.
Astrophysical Journal, 8, p. 48, 1898.
Preston. *Proc. Royal Soc.*, 63, p. 26, 1898.
Phil. Mag., (5), 45, p. 325, 1898; 47, p. 165, 1899.
Phil. Mag., 45, p. 197, 1899.
Zeeman. *Astrophysical Journal*, 9, p. 47, 1899.
Proc. Royal Ac. Amsterdam, Dec., 1899.
Reese. *Johns Hopkins University Circulars*, 18, p. 59, 1899.
W. König. *Wied. Ann.*, 63, p. 268, 1899.
A. Cotton. *Le phénomène de Zeeman* (Scientia), George Carré & C. Nand, Paris, 1899.
W. Voigt. *Physikalische Zeitschrift*, No. 10, p. 116; No. 11, p. 128; No. 12, p. 138, 1899.
Phil. Mag., (5), 48, p. 317, 1899.

Theoretical

H. A. Lorentz. *Wied. Ann.*, 63, p. 278, 1897.
Arch. Néerl., 2, p. 1, 1898.
Astrophysical Journal, 9, p. 37, 1899.
Proc. Royal Ac. Amsterdam, June, 1899.
Larmor. *Proc. Royal Soc.*, 60, p. 514, 1897.
Phil. Mag., (5), 44, p. 503, 1897.
Leathem. *Proc. Royal Soc.*, 61, p. 487, 1897.
Trans. Cambridge Phil. Soc., 17, p. 16, 1898.
Fitzgerald. *Proc. Royal Soc.*, 63, p. 31, 1898.
W. Voigt. *Göttinger Nachrichten*, Heft 4, 1898.
Wied. Ann., 67, p. 345–366, 1899; 68, p. 352, 1899; 69, p. 290, 1899. *Drude's Ann.*, 1, p. 389, 1900.
Macaluso and Corbino. *Rendic. R. Ac. dei Lincei*, vol. 8, ser. 9, p. 116, 1899.
Corbino. *Rendic. R. Ac. dei Lincei*, vol. 8, p. 172, 1899; p. 250, 1899.
Goldhammer. *Wied. Ann.*, 67, p. 697, 1898.
Poincaré. *Éclairage Électrique*, t. 19, p. 5, 1899.
Righi. *Physikalische Zeitschrift*, No. 30, p. 329, 1900.

INDEX

INDEX

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